

# On Rationality Conditions for Epistemic Probabilities in Abstract Argumentation

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**Abstract.** Epistemic probabilities in argumentation frameworks are meant to represent subjective degrees of belief in the acceptance of arguments. As such, they are subject to some rationality conditions, taking into account the attack relation between arguments. This paper provides an advancement with respect to the previous literature on this matter by casting epistemic probabilities in the context of de Finetti's theory of subjective probability and by analyzing and revising the relevant rationality properties in relation with de Finetti's notion of *coherence*. Further, we consider the extension to Walley's theory of imprecise probabilities and carry out a preliminary analysis about rationality conditions in this more general context.

**Keywords.** Argumentation frameworks, Epistemic probabilities, Coherent probabilities

## Introduction

The use of probabilistic evaluations in abstract argumentation frameworks has been receiving an increasing attention in recent years [8,9,4,5,6,7]. As discussed in [6] two main approaches can be identified: briefly, in the *constellations* approach there is uncertainty about the actual framework to be considered, while in the *epistemic* approach a fixed framework is considered and uncertainty concerns the acceptance (rather than the existence) of arguments in the framework. In the latter context, some rationality conditions on probability assignments can be introduced, taking into account the attack relation encoded by the framework. In particular, the notion of *p-justifiable* probability function is introduced in [9] while *rational* and *coherent* probability functions are considered in [6]. While epistemic probabilities are claimed to have a *subjective* nature in [9], the traditional Kolmogorov axioms have been adopted rather than de Finetti's subjective probability theory [2]. However, the latter appears to be more appropriate in this context.

This paper contributes to the advancement of this investigation line by casting epistemic probabilities in the context of de Finetti's theory, revising and discussing rationality conditions for epistemic probabilities accordingly, and then extending the analysis to the more general case of imprecise rather than precise probability evaluations. The paper is organised as follows. Section 1 provides the necessary background on argumentation frameworks and on precise and imprecise subjective probabilities. Section 2 discusses precise epistemic probabilities and their rationality conditions, while Section 3 deals with the case of imprecise assessments. Finally, Section 4 concludes the paper.

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## 1. Background

### 1.1. Argumentation frameworks

We quickly recall the basic notions of Dung’s abstract argumentation frameworks [3].

**Definition 1** An argumentation framework ( $AF$ ) is defined as a pair  $\langle \mathcal{A}, \rightarrow \rangle$  in which  $\mathcal{A}$  is a set of arguments and  $\rightarrow \subseteq \mathcal{A} \times \mathcal{A}$  describes the attack relation between arguments in  $\mathcal{A}$ , so that  $(\alpha, \beta) \in \rightarrow$  (also denoted as  $\alpha \rightarrow \beta$ ) indicates that the argument  $\alpha$  attacks the argument  $\beta$  (while we will denote  $(\alpha, \beta) \notin \rightarrow$  as  $\alpha \not\rightarrow \beta$ ). For a set  $S \subseteq \mathcal{A}$ , the attackers of  $S$  are defined as  $S^- = \{\alpha \in \mathcal{A} \mid \exists \beta \in S : \alpha \rightarrow \beta\}$  and the attackees of  $S$  are defined as  $S^+ = \{\alpha \in \mathcal{A} \mid \exists \beta \in S : \beta \rightarrow \alpha\}$ . An argument  $\alpha$  is self-defeating if  $\alpha \rightarrow \alpha$ .

Building on the attack relation, the basic concepts of *conflict-freeness*, *defense* (called *acceptability* in [3]), and *reinstatement* can be introduced.

**Definition 2** Given an  $AF = \langle \mathcal{A}, \rightarrow \rangle$ , a set of arguments  $S \subseteq \mathcal{A}$  is conflict-free, denoted as  $S \in \mathcal{E}_{CF}(AF)$ , iff  $S \cap S^+ = \emptyset$ . An argument  $\alpha$  is defended by a set  $S \subseteq \mathcal{A}$  (or  $\alpha$  is acceptable w.r.t  $S$ ) iff  $\{\alpha\}^- \subseteq S^+$ . A set of arguments  $S$  satisfies the reinstatement property iff  $S$  contains all the arguments it defends.

Given an  $AF$ , encoding the conflicts in a set of arguments, a fundamental problem consists in determining the conflict outcome, namely assigning a justification status to arguments. An argumentation semantics can be conceived, in broad terms, as a formal way to answer this question. More precisely, in the *extension-based* approach an argumentation semantics specifies the criteria for identifying, for a generic  $AF$ , a set of *extensions*, where each extension is a set of arguments considered to be jointly acceptable. Dealing with different argumentation semantics (see [1] for a review) is not necessary for the purposes of this paper: it is sufficient to recall that all literature semantics share the very basic property that their extensions are conflict-free and most of them satisfy the reinstatement property too.

### 1.2. Precise and imprecise subjective probabilities

#### 1.2.1. Events

Following de Finetti’s theory [2], a subjective probability assessment  $P$  expressed by an agent  $\mathbf{A}$  is a function  $P : E \rightarrow \mathbb{R}$ , where  $E$  is an *arbitrary* (finite or infinite) set of events. Therefore, in order to characterize subjective probability assessments we need first to introduce the notion of event and to discuss the fact that, unlike traditional probability theory, no requirements on the set of events  $E$  are given.

An event is represented by a statement<sup>2</sup> that can be either **true** or **false** and whose actual truth value is (typically) unknown to the agent at the moment of the assessment. The conjunction (or logical product, denoted as  $\wedge$ ), disjunction (or logical sum, denoted as  $\vee$ ), and negation (or complement, denoted as  $\neg$ ) of events are events too and have the usual meanings. Moreover, an event  $e_i$  implies an event  $e_j$  (denoted as  $e_i \Rightarrow e_j$ )

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<sup>2</sup>In general, the same event can be represented by different but logically equivalent statements. Discussing this and other related aspects is beyond the scope of this paper.

if it can not be the case that  $e_j$  is **false** and  $e_i$  is **true**. Two events  $e_i, e_j$  are equal (denoted as  $e_i \equiv e_j$ ) when both  $e_i \Rightarrow e_j$  and  $e_j \Rightarrow e_i$ . Two special events are the *certain event*  $\top$ , which is always **true**, and the *impossible event*  $\perp$ , which is always **false**.

A probability assessment concerns an *arbitrary* set of events  $E$ , containing all and only the events that are of some interest for the agent. As such, the events in  $E$  need not satisfy any specific property, such as being pairwise disjoint or exhaustive. Formally it may be the case that for some  $e_i, e_j \in E$ ,  $e_i \wedge e_j \not\equiv \perp$ , and that  $\bigvee_{e_i \in E} e_i \not\equiv \top$ .

As an example, consider a football match involving a team  $T$ : one may be interested in assessing the probability of the events  $e_1 = \text{“}T \text{ gets at least one point”}$  and  $e_2 = \text{“}T \text{ wins the match”}$ . Clearly these events are not pairwise disjoint, since  $e_2 \Rightarrow e_1$ , and the set  $E = \{e_1, e_2\}$  is not exhaustive since  $e_1 \vee e_2$  does not cover the case where  $T$  loses.

A set of pairwise disjoint (non-impossible) events whose logical sum is the certain event is called a *partition*. We use the symbol  $\Omega$  (possibly with superscripts) to denote a partition, whose events  $\omega_i$  are called the *atoms* of  $\Omega$ . In the football match example, a possible partition would be  $\Omega = \{\omega_1, \omega_2, \omega_3\}$ , where  $\omega_1 = \text{“}T \text{ loses”}$ ,  $\omega_2 = \text{“}T \text{ draws”}$ ,  $\omega_3 = \text{“}T \text{ wins”}$ . Note that the same “phenomenon” can be described by different partitions, for instance by  $\Omega^1 = \{\omega_1^1, \omega_2^1\}$ , where  $\omega_1^1 = \text{“}T \text{ scores no goals”}$  and  $\omega_2^1 = \text{“}T \text{ scores at least one goal”}$ . While in this example there are very limited relations between the atoms of the two partitions (the only one being that  $\omega_1^1$  is incompatible with  $\omega_3$ , i.e.  $\omega_1^1 \wedge \omega_3 \equiv \perp$ ), it is interesting to consider the case where two partitions are strongly related since one partition is a refinement of the other.

Formally, a partition  $\Omega^f$  is *more refined* than a partition  $\Omega$  (or, equivalently,  $\Omega$  is coarser than  $\Omega^f$ ) iff every atom of  $\Omega$  is the logical sum of atoms of  $\Omega^f$ . Continuing our example, consider the partition  $\Omega^2 = \{\omega_1^2, \omega_2^2, \omega_3^2, \omega_4^2, \omega_5^2\}$  where  $\omega_1^2 = \text{“}T \text{ loses by one goal”}$ ,  $\omega_2^2 = \text{“}T \text{ loses by more than one goal”}$ ,  $\omega_3^2 = \text{“}T \text{ draws”}$ ,  $\omega_4^2 = \text{“}T \text{ wins by one goal”}$ ,  $\omega_5^2 = \text{“}T \text{ wins by more than one goal”}$ . Then  $\Omega^2$  is more refined than the partition  $\Omega$  introduced above, since  $\omega_1 \equiv (\omega_1^2 \vee \omega_2^2)$ ,  $\omega_2 \equiv \omega_3^2$ , and  $\omega_3 \equiv (\omega_4^2 \vee \omega_5^2)$ .

Note that, apart from very simple settings, in general an endless refinement process can be considered (e.g. in the football example, one could consider finer distinctions concerning the number of goals scored, the identity of the scoring players, the scoring times, and so on): the choice of the reference partition only depends on the current agent needs and interests (which of course may vary over time).

A set of events  $E$  can be defined without reference to any partition. However, for any  $E$  the partition generated by  $E$ , denoted as  $\mathcal{P}_g(E)$ , is defined as the set of all the non-impossible logical products of the form  $\bigwedge_{e_i \in E} e'_i$ , where each  $e'_i$  is alternatively replaced by either  $e_i$  or its negation  $\neg e_i$ . Coming back to the beginning of our example, given  $E = \{e_1, e_2\}$ , the four candidate atoms for  $\mathcal{P}_g(E)$  are  $e_1 \wedge e_2$ ,  $e_1 \wedge \neg e_2$ ,  $\neg e_1 \wedge e_2$ ,  $\neg e_1 \wedge \neg e_2$ . Of these,  $\neg e_1 \wedge e_2$  is impossible since it can not be the case that a team wins ( $e_2$ ) without getting any point ( $\neg e_1$ ). The remaining three candidates are the atoms of  $\mathcal{P}_g(E) = \{e_1 \wedge e_2, e_1 \wedge \neg e_2, \neg e_1 \wedge \neg e_2\}$ .

For any set  $E$ ,  $\mathcal{P}_g(E)$  satisfies the following properties: (i) every event  $e_i$  of  $E$  is the logical sum of some atoms of  $\mathcal{P}_g(E)$  (those implying  $e_i$ ); (ii)  $\mathcal{P}_g(E)$  is the coarsest partition with the property (i).

### 1.2.2. Coherence conditions for precise and imprecise probabilities

Given an arbitrary set of events  $E$ , in subjective probability theory a (precise) probability assessment  $P : E \rightarrow \mathbb{R}$  by an agent  $\mathbf{A}$  is interpreted in the context of an idealized betting

scheme, where, for each event  $e$ ,  $P(e)$  is the “fair” price of a (unitary) bet on  $e$ , i.e.  $P(e)$  is the amount of money that **A** is ready to pay to an opponent **O** in order to receive from **O** the sum of 1 if  $e$  turns out to be **true** and 0 otherwise, and, indifferently, the sum that **A** is ready to receive from **O** as a payment for the commitment to pay to **O** the sum of 1 if  $e$  turns out to be **true** and 0 otherwise. More formally,  $P(e)$  is the price, according to **A**, of the *indicator* of  $e$ , denoted as  $I(e)$ , namely the random number which takes value 1 if  $e$  is **true**, and value 0 if  $e$  is **false**. It is assumed that **A** is indifferently ready to buy or sell  $I(e)$  at price  $P(e)$ . In the case of buying, the random gain of **A** is  $I(e) - P(e)$ , while it is  $P(e) - I(e)$  in the case of selling. A not necessarily unitary bet is characterized by a real coefficient (or stake)  $s \in \mathbb{R}$ , so that the gain of **A** is given by  $s(I(e) - P(e))$ . A positive (negative) value of  $s$  corresponds to a buying (selling) choice by **A**.

According to the betting interpretation, a probability assessment has to satisfy some conditions ensuring that the bet makes sense for both participants. In particular, de Finetti has established a property of coherence, called *dF-coherence* in the sequel.

**Definition 3** *Given an arbitrary set of events  $E$ ,  $P : E \rightarrow \mathbb{R}$  is a dF-coherent probability if and only if  $\forall n \in \mathbb{N}^+, \forall s_1, \dots, s_n \in \mathbb{R}, \forall e_1, \dots, e_n \in E$ , it holds that*

$$\max \left[ \sum_{i=1}^n s_i (I(e_i) - P(e_i)) \right] \geq 0 \quad (1)$$

where  $\mathbb{N}^+$  is the set of positive integer numbers.

Intuitively dF-coherence states that for any finite combination of bets, the maximum value of the random gain of **A** is non-negative, hence **A** avoids a sure loss. Among the many properties of dF-coherent probabilities, we recall the following ones for later use.

**Proposition 1** *Let  $P : E \rightarrow \mathbb{R}$  be a dF-coherent probability. Then:*

- a) *if  $\top \in E$ ,  $P(\top) = 1$ ; if  $\perp \in E$ ,  $P(\perp) = 0$  (normalisation)*
- b) *if  $e_1, e_2 \in E$  and  $e_1 \Rightarrow e_2$  then  $P(e_1) \leq P(e_2)$  (monotonicity)*
- c)  *$\forall e \in E$ ,  $0 \leq P(e) \leq 1$  (non negativity)*
- d)  *$\forall e_1, e_2, e_1 \wedge e_2 \in E$ ,  $P(e_1 \vee e_2) = P(e_1) + P(e_2) - P(e_1 \wedge e_2)$*
- e) *if  $E$  is a finite partition, i.e.  $E = \Omega = \{\omega_1, \dots, \omega_n\}$ ,  $P$  is dF-coherent on  $E$  iff  $\forall i = 1, \dots, n$   $P(\omega_i) \geq 0$  and  $\sum_{i=1}^n P(\omega_i) = 1$ .*

Note in particular that c) is implied by a) and b), because  $\forall e \perp \Rightarrow e \Rightarrow \top$ . Observe also that, for any event  $e$ , a) and d) imply that

$$P(e) = 1 - P(\neg e) \quad (\text{self-conjugacy}) \quad (2)$$

The properties above are necessary for dF-coherence of  $P$ , but generally not sufficient. A useful way to detect whether a map  $P$  on a *finite* set of events  $E$  is dF-coherent is suggested by Proposition 1.e) and the following Property 1.

**Property 1** *A map  $P : E \rightarrow \mathbb{R}$ , where  $E$  is a finite but arbitrary set of events, is a dF-coherent probability if and only if there is a dF-coherent probability  $\tilde{P} : \mathcal{P}_g(E) \rightarrow \mathbb{R}$  such that  $\forall e \in E$   $P(e) = \sum_{\omega \in \mathcal{P}_g(E): \omega \Rightarrow e} \tilde{P}(\omega)$ .*

Another important property of dF-coherent probabilities is the possibility to extend a “partial” assessment on a set of events to a larger set of events preserving both the probability values already given and the coherence property. This is established by the following *extension theorem*.

**Theorem 1** *Given a dF-coherent probability  $P$  on a set of events  $E$ , and  $E' \supseteq E$ , there is a dF-coherent probability  $P'$  on  $E'$  such that  $\forall e \in E P'(e) = P(e)$ .*

Imprecise probabilities [10] can be introduced, in a betting scheme context, by lifting the assumption that the agent  $\mathbf{A}$  has a precise price estimation, used indifferently for buying or selling event indicators. Rather (as typical in real markets)  $\mathbf{A}$  considers, for each event  $e$ , two different prices, one for buying and one for selling  $I(e)$ , denoted respectively as  $\underline{P}(e)$  and  $\overline{P}(e)$ . Clearly,  $\underline{P}(e) \leq \overline{P}(e)$ . Moreover,  $\mathbf{A}$  is of course ready to buy also at any price lesser than  $\underline{P}(e)$ , which hence represents the *supremum buying price* for  $I(e)$ . Similarly,  $\overline{P}(e)$  is the *infimum selling price* for  $I(e)$ . Given that, for any event  $e$ ,  $I(-e) = 1 - I(e)$ , it turns out that buying an event is equivalent to selling its complement and vice versa. Hence, in the context of imprecise probabilities, the following *conjugacy relation* replaces condition (2):

$$\underline{P}(e) = 1 - \overline{P}(-e) \quad (3)$$

In virtue of the conjugacy relation, one can focus on lower (as we will mostly do in the following) or upper probabilities only.

Definition 4 provides the notion of coherence for lower probabilities [10].

**Definition 4** *Given an arbitrary set of events  $E$ ,  $\underline{P} : E \rightarrow \mathbb{R}$  is a coherent lower probability if and only if  $\forall n \in \mathbb{N} = \mathbb{N}^+ \cup \{0\}$ , and for all real and non-negative  $s_0, \dots, s_n$ ,  $\forall e_0, \dots, e_n \in E$ , it holds that*

$$\max \left[ \left[ \sum_{i=1}^n s_i (I(e_i) - \underline{P}(e_i)) \right] - s_0 (I(e_0) - \underline{P}(e_0)) \right] \geq 0 \quad (4)$$

The coherence condition requires that the maximum of the gain of  $\mathbf{A}$  is non negative for every (including the empty) combination of buying bets with at most one selling bet of a single (arbitrarily selected) event  $e_0$ . In a sense Definition 4 allows  $\mathbf{A}$  to use its supremum buying price for any buying transaction but also forces  $\mathbf{A}$  to use the same price for (at most one) selling transaction. Intuitively, this ensures that the assessment  $\underline{P}$  by  $\mathbf{A}$  is not too unfair. Among the properties of coherent imprecise probabilities, the analogues of a), b), c) of Proposition 1 hold (just replace there  $P$  with either  $\underline{P}$  or  $\overline{P}$ ). Note that the *vacuous* (non-informative) lower probability  $\underline{P}(e) = 0 \forall e \neq \top$  (or equivalently the upper probability  $\overline{P}(e) = 1 \forall e \neq \perp$ ) is coherent for every set of events.

Coherent imprecise probabilities are related to precise probabilities by the lower envelope theorem ([10] Sec. 3.3.3).

**Theorem 2** *Given an arbitrary set of events  $E$ ,  $\underline{P} : E \rightarrow \mathbb{R}$  is a coherent lower probability if and only if there is a set  $P_\Gamma = \{P_\gamma \mid \gamma \in \Gamma\}$  of dF-coherent probabilities  $P_\gamma$  on  $E$  such that  $\forall e \in E, \underline{P}(e) = \min_{\gamma \in \Gamma} P_\gamma(e)$ .*

In words, a lower probability  $\underline{P}$  is coherent if and only if it can be obtained as the lower envelope of a set  $(P_\Gamma)$  of dF-coherent precise probabilities  $(P_\gamma)$ . This result provides both a constructive procedure for coherent lower probabilities and a motivation for their existence: when different probability assessments are at hand (e.g. given by different experts or because we are unsure about the “true” probability) coherent lower probabilities arise by aggregating them in the least committed way. Finally, it has to be mentioned that an extension theorem holds for imprecise probabilities too ([10] Sec. 3.1). Its detailed description is out of the scope of this paper.

## 2. Epistemic probabilities in argumentation frameworks

An epistemic probability function  $P$  is simply the assignment of a (precise) probability value to each argument of a framework [9,6].

**Definition 5** *A probabilistic argumentation framework (PAF) is a triple  $\langle \mathcal{A}, \rightarrow, P \rangle$  where  $\langle \mathcal{A}, \rightarrow \rangle$  is an argumentation framework and  $P : \mathcal{A} \rightarrow [0, 1]$ .*

Different origins for probability values can be considered: in [9] probabilities of individual arguments are derived from an initial assignment on sets of arguments, while in [6] probabilities of abstract arguments derive from probabilities of instantiated arguments in a logic-based argumentation system. Independently of these differences, epistemic probabilities are meant to express a *degree of belief* held by an agent about each argument: the higher the belief degree the higher the propensity of the agent to accept an argument. In this perspective, it can be required that the agent beliefs are not in contrast with the attack relation between arguments: intuitively, an agent should not be inclined to accept at the same time two arguments which are in conflict.

This basic intuition is formally expressed by some *rationality* properties of probabilistic argumentation frameworks, recalled in the following definitions.

**Definition 6** *(Rational probability [6]) Given a PAF  $= \langle \mathcal{A}, \rightarrow, P \rangle$ ,  $P$  is rational iff for every pair  $(\alpha, \beta)$  such that  $\beta \rightarrow \alpha$  if  $P(\beta) > 0.5$  then  $P(\alpha) \leq 0.5$ .*

**Definition 7** *(p-justifiable probability [9]) Given a PAF  $= \langle \mathcal{A}, \rightarrow, P \rangle$ ,  $P$  is p-justifiable iff for every  $\alpha \in \mathcal{A}$  the following conditions hold:*

1.  $\forall \beta \in \{\alpha\}^- P(\alpha) \leq 1 - P(\beta)$ ;
2.  $P(\alpha) \geq 1 - \sum_{\beta \in \{\alpha\}^-} P(\beta)$ .

Clearly, the rationality property of Def. 6 is implied by condition 1 of Def. 7, hence we will focus our analysis on p-justifiability. Looking first at its intuitive interpretation, we observe that condition 1 can be directly related to conflict freeness (consider in particular its equivalent reading  $P(\alpha) + P(\beta) \leq 1$  for all  $(\alpha, \beta) \in \rightarrow$ ). Condition 2 can instead be related to reinstatement: if all its attackers have collectively a low degree of belief, an argument can not have a low degree of belief too. As a special case, note that if an argument  $\alpha$  has no attackers (or all its attackers have zero probability) then by condition 2,  $P(\alpha) = 1$ . It can also be observed that, in case  $\alpha$  has exactly one attacker  $\beta$ , conditions 1 and 2 imply that  $P(\alpha) = 1 - P(\beta)$ .

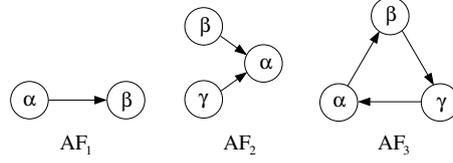


Figure 1. Some simple argumentation frameworks.

In order to relate epistemic probabilities with the theory of coherent probabilities we need first to characterize the set of events the probability assessment refers to. Since epistemic probabilities concern the acceptance of arguments, in the following we denote the event “ $\alpha$  is accepted” as  $\hat{\alpha}$ , and the probability that  $\alpha$  is accepted as  $P(\hat{\alpha})$ . The notion of acceptance is considered as having a binary nature, basically coinciding with membership in the (crisp) set of accepted arguments. Given a set of arguments  $S$ , we define  $E_{ac}(S) = \{\hat{\alpha} \mid \alpha \in S\}$  and  $E_{\neg ac}(S) = \{\neg\hat{\alpha} \mid \alpha \in S\}$ .

It can then be observed that given an argumentation framework  $AF = \langle \mathcal{A}, \rightarrow \rangle$  with  $\mathcal{A} = \{\alpha_1, \dots, \alpha_n\}$  the events in the set  $E_{ac}(\mathcal{A}) = \{\hat{\alpha}_1, \dots, \hat{\alpha}_n\}$  are generally not mutually exclusive, since it may well be the case that  $\alpha_i$  and  $\alpha_j$ ,  $i \neq j$ , are accepted at the same time, nor exhaustive, since  $\bigwedge_{i=1}^n \neg\hat{\alpha}_i$  is always possible.

Hence  $E_{ac}(\mathcal{A})$  is not a partition, but, as discussed in Section 1.2.1, the generated partition  $\mathcal{P}_g(E_{ac}(\mathcal{A}))$  can be considered whose candidate events have the form  $\hat{\alpha}_1' \wedge \dots \wedge \hat{\alpha}_n'$ . The next step is to identify which of them are impossible. Given the intuitive meaning of the attack relation, the fact that conflict-freeness is the most basic property shared by all semantics, and, last but not least, the intuition underlying the rationality conditions in [9] and [6], it is natural to regard as impossible those events where both  $\hat{\alpha}_i$  and  $\hat{\alpha}_j$  are asserted and  $\alpha_i \rightarrow \alpha_j$  holds in  $AF$ . This is summarized by Definition 8.

**Definition 8** Given an argumentation framework  $AF = \langle \mathcal{A}, \rightarrow \rangle$ , let  $S \subseteq \mathcal{A}$  be a conflict free set. The acceptance event corresponding to  $S$  is defined as  $ae(S) = \bigwedge_{\alpha_i \in S} \hat{\alpha}_i \wedge \bigwedge_{\alpha_j \in \mathcal{A} \setminus S} \neg\hat{\alpha}_j$ . The partition of acceptance events generated by  $AF$  is defined as  $\mathcal{P}_g^{AF} = \{ae(S) \mid S \in \mathcal{E}_{CF}(AF)\}$ .

As an example, consider the argumentation framework  $AF_1 = \langle \{\alpha, \beta\}, \{(\alpha, \beta)\} \rangle$  in Figure 1. Then  $\mathcal{P}_g^{AF_1} = \{\hat{\alpha} \wedge \neg\hat{\beta}, \neg\hat{\alpha} \wedge \hat{\beta}, \neg\hat{\alpha} \wedge \neg\hat{\beta}\}$ ; later  $p_1, p_2, p_3$  will denote the respective probability values of these atoms.

We can now analyze the relationships between conditions 1 and 2 of Definition 7 and the dF-coherence of precise probabilities.

Proposition 2 ensures that condition 1 is implied by dF-coherence of  $P$ .

**Proposition 2** Let  $AF = \langle \mathcal{A}, \rightarrow \rangle$  be an argumentation framework. If  $P$  is a dF-coherent precise probability on  $E_{ac}(\mathcal{A})$  then  $P$  satisfies condition 1 of Definition 7.

**Proof:** Given  $\alpha$  and  $\beta$  such that  $\alpha \rightarrow \beta$ , by the requirement of conflict freeness we have that  $\hat{\beta} \Rightarrow \neg\hat{\alpha}$ . Applying Proposition 1.b) and (2) we obtain  $P(\hat{\beta}) \leq P(\neg\hat{\alpha}) = 1 - P(\hat{\alpha})$ , which is condition 1.  $\square$

Adding condition 2 to condition 1, in the  $AF_1$  case, gives rise to  $P(\hat{\beta}) = 1 - P(\hat{\alpha})$ , i.e., by Property 1, to  $p_2 = p_2 + p_3$ , hence  $p_3 = 0$ , which corresponds to exclude the case where both  $\alpha$  and  $\beta$  are rejected and is consistent with the reinstatement principle.

Note that the same conclusion can be obtained also by considering condition 2 alone, since  $P(\hat{\beta}) \geq 1 - P(\hat{\alpha})$  leads to  $p_2 \geq p_2 + p_3 \Leftrightarrow p_3 = 0$ , using Proposition 1.e). Hence, condition 2 appears to be a rather strong requirement even in this simple case.

Let us now consider condition 2 in the case of an argument with two attackers:  $AF_2 = \langle \{\alpha, \beta, \gamma\}, \{(\beta, \alpha), (\gamma, \alpha)\} \rangle$  in Figure 1. Here,  $\mathcal{P}_g^{AF_2} = \{\hat{\alpha} \wedge \neg \hat{\beta} \wedge \neg \hat{\gamma}, \neg \hat{\alpha} \wedge \hat{\beta} \wedge \hat{\gamma}, \neg \hat{\alpha} \wedge \neg \hat{\beta} \wedge \hat{\gamma}, \neg \hat{\alpha} \wedge \hat{\beta} \wedge \neg \hat{\gamma}, \neg \hat{\alpha} \wedge \neg \hat{\beta} \wedge \neg \hat{\gamma}\}$ , and letting  $p_1, \dots, p_5$  be the respective probability values we get  $P(\hat{\alpha}) = p_1$ ,  $P(\hat{\beta}) = p_2 + p_4$ ,  $P(\hat{\gamma}) = p_2 + p_3$ .

Condition 2 then reads as  $P(\hat{\alpha}) \geq 1 - (P(\hat{\beta}) + P(\hat{\gamma}))$  i.e.  $p_1 \geq 1 - (p_2 + p_4 + p_2 + p_3)$  which, using  $\sum_{i=1}^5 p_i = 1$ , reduces to  $p_2 \geq p_5$ . This constraint might be interpreted as a form of weak reinstatement (accepting  $\beta$  and  $\gamma$  is more likely than rejecting both of them), however a further consideration can be made. Actually, condition 2 can be thought of as arising from the requirement that the probability of (acceptance of) an argument is not lower than the complement of the probability that at least one of its attackers is accepted, i.e. of the disjunction of (the acceptance of) its attackers. However, the probability of the disjunction of the attackers is equal to the sum of the probabilities of acceptance of each attacker only in the case the relevant acceptance events are incompatible (i.e. each attacker is in conflict with every other attacker). This is not the case in general (and in particular not in  $AF_2$ ). Accordingly, we propose a modification of p-justifiability, called *revised p-justifiability* (rp-justifiability) where condition 2 of Def. 7 is replaced by:

$$P(\hat{\alpha}) \geq 1 - P\left(\bigvee_{\beta \in \{\alpha\}^-} \hat{\beta}\right) \quad (5)$$

In order to apply rp-justifiability to  $AF_2$ , first note that  $P(\hat{\beta} \vee \hat{\gamma}) = P(\hat{\beta}) + P(\hat{\gamma}) - P(\hat{\beta} \wedge \hat{\gamma})$ . It follows  $P(\hat{\beta} \vee \hat{\gamma}) = p_2 + p_4 + p_2 + p_3 - p_2$ , and condition (5) reads as  $p_1 \geq 1 - (p_2 + p_4 + p_3)$ , from which, using again  $\sum_{i=1}^5 p_i = 1$ , it follows  $p_5 = 0$ . Note that the result obtained from the new condition (5) is in accordance with the one obtained for  $AF_1$  (where the original and the new condition coincide): according to the reinstatement principle, the case where all arguments are rejected is given zero probability.

In general, p- and rp-justifiability coincide when each argument has exactly one attacker. In this case, the two relevant constraints reduce to:

$$\forall \beta \in \{\alpha\}^- P(\alpha) = 1 - P(\beta). \quad (6)$$

A probability assessment satisfying condition (6) is called *involutory* in [9]. As also observed in [6], in the case of an  $AF$  consisting of an odd-length cycle, there can be no probability assessment satisfying condition (6), as exemplified by the argumentation framework  $AF_3 = \langle \{\alpha, \beta, \gamma\}, \{(\alpha, \beta), (\beta, \gamma), (\gamma, \alpha)\} \rangle$  in Figure 1.

Here,  $\mathcal{P}_g^{AF_3} = \{\hat{\alpha} \wedge \neg \hat{\beta} \wedge \neg \hat{\gamma}, \neg \hat{\alpha} \wedge \hat{\beta} \wedge \neg \hat{\gamma}, \neg \hat{\alpha} \wedge \neg \hat{\beta} \wedge \hat{\gamma}, \neg \hat{\alpha} \wedge \neg \hat{\beta} \wedge \neg \hat{\gamma}\}$ . Letting  $p_1, \dots, p_4$  be the respective probability values we get the following three equations  $p_1 = 1 - p_3$ ,  $p_3 = 1 - p_2$ ,  $p_2 = 1 - p_1$ , whose only solution  $p_1 = p_2 = p_3 = 0.5$  is clearly not a dF-coherent probability assessment, since it violates Proposition 1.e). This contradicts the claim of Proposition 3 of [9] that the set of p-justifiable probabilities is always non-empty, and shows that also rp-justifiability runs into the same difficulty.

Since the problem originates essentially from condition 2 of Definition 7 one can consider weakening the notion of p-justifiability by suppressing condition 2 and reformulating condition 1.

**Definition 9** Given a PAF  $\langle \mathcal{A}, \rightarrow, P \rangle$ ,  $P$  is weakly p-justifiable iff for every  $\alpha \in \mathcal{A}$  the following condition hold:

$$\forall \beta \in \{\alpha\}^- \quad P(\hat{\alpha}) \leq 1 - P(\hat{\beta}) \quad (7)$$

The question of the existence of at least one *weakly p-justifiable* probability for every argumentation framework has a rather trivial answer.

**Proposition 3** Given an argumentation framework  $AF = \langle \mathcal{A}, \rightarrow \rangle$  there is at least one PAF  $\langle \mathcal{A}, \rightarrow, P \rangle$  such that  $P$  is dF-coherent and weakly p-justifiable.

**Proof:** For every  $AF$ , the event  $\phi_{AF} \triangleq \bigwedge_{\alpha \in \mathcal{A}} \neg \hat{\alpha}$  belongs to  $\mathcal{P}_g^{AF}$ . Letting  $P(\phi_{AF}) = 1$ , we get  $P(\hat{\alpha}) = 0$  for every argument  $\alpha$ . Then for every pair  $\alpha, \beta$  with  $\beta \in \{\alpha\}^-$  condition (7) reduces to  $0 \leq 1$  and is trivially verified.  $\square$

The simple probability assignment considered in the proof of Proposition 3 is the only possible one in the case all arguments are self-defeating. In this case  $\mathcal{E}_{\mathcal{CF}}(AF) = \{\emptyset\}$  and  $\phi_{AF}$  is the only non-impossible event, i.e. the only member of  $\mathcal{P}_g^{AF}$ . Leaving apart this degenerate case, it can be shown that there is always a weakly p-justifiable probability with non zero probability on those arguments which are not self-defeating.

**Proposition 4** Given an argumentation framework  $AF = \langle \mathcal{A}, \rightarrow \rangle$  such that  $\mathcal{E}_{\mathcal{CF}}(AF) \neq \{\emptyset\}$ , there is at least one PAF  $\langle \mathcal{A}, \rightarrow, P \rangle$  such that  $P$  is dF-coherent, weakly p-justifiable and  $P(\hat{\alpha}) > 0$  for any  $\alpha$  such that  $\alpha \not\rightarrow \alpha$ .

**Proof:** Clearly  $\mathcal{P}_g^{AF} \supseteq \{\phi_{AF}\}$ . Let  $\mathcal{P}_g^{AF} = \{\phi_{AF}, \omega_1, \dots, \omega_p\}$  with  $p \geq 1$  and consider the assignment on  $\mathcal{P}_g^{AF}$ :  $P(\phi_{AF}) = 0.5$ ,  $P(\omega_i) = 1/(2 \cdot p) > 0$  for  $i = 1, \dots, p$ . Then, for every  $\alpha \in \mathcal{A}$ ,  $P(\hat{\alpha}) = \sum_{\omega_i \Rightarrow \hat{\alpha}} P(\omega_i) \leq 0.5$ . Noting that the set of atoms implying  $\hat{\alpha}$  is not empty if  $\alpha \not\rightarrow \alpha$ , it follows that  $0 < P(\hat{\alpha}) \leq 0.5$  for every non self-defeating argument  $\alpha$ . This implies in particular  $P(\hat{\alpha}_i) \leq 1 - P(\hat{\alpha}_j)$  for every  $i, j$ .  $\square$

### 3. Extending epistemic probabilities to imprecise assessments

In this section we extend our analysis to imprecise probabilities. Doing so, we take the opportunity to consider a further generalisation concerning the set of events whose probability is assessed. In particular, while in Section 2, following [9], we assumed that, given  $AF = \langle \mathcal{A}, \rightarrow \rangle$ , a precise probability is given for all the events in  $E_{ac}(\mathcal{A})$ , here we consider assessments on arbitrary sets of acceptance or non-acceptance events. More formally, given  $AF = \langle \mathcal{A}, \rightarrow \rangle$ , we consider lower (upper) probability assessments  $\underline{P}$  ( $\overline{P}$ ) on  $E \subseteq E_{ac}(\mathcal{A}) \cup E_{\neg ac}(\mathcal{A})$ . Accordingly, the rationality conditions will be required to hold whenever the relevant events belong to  $E$ .

First, we need to extend the notion of weak p-justifiability to the case of imprecise probabilities. To this purpose note that, in the case of precise probabilities, weak p-justifiability has two equivalent formulations, letting  $\beta \in \{\alpha\}^-$ :

1.  $P(\hat{\alpha}) \leq 1 - P(\hat{\beta})$ ;
2.  $P(\hat{\alpha}) \leq P(\neg \hat{\beta})$ .

With lower, rather than precise, probabilities the first inequality translates to

$$(i) \underline{P}(\hat{\alpha}) \leq 1 - \underline{P}(\hat{\beta}) \Leftrightarrow \underline{P}(\hat{\alpha}) \leq \overline{P}(\neg\hat{\beta}),$$

the second to (ii)  $\underline{P}(\hat{\alpha}) \leq \underline{P}(\neg\hat{\beta})$ .

Similarly with upper probabilities the first formulation is

$$(iii) \overline{P}(\hat{\alpha}) \leq 1 - \overline{P}(\hat{\beta}) \Leftrightarrow \overline{P}(\hat{\alpha}) \leq \underline{P}(\neg\hat{\beta}),$$

while the second is (iv)  $\overline{P}(\hat{\alpha}) \leq \overline{P}(\neg\hat{\beta})$ .

Using  $\underline{P}(e) \leq \overline{P}(e)$ , one can note that (iii)  $\Rightarrow$  (ii)  $\Rightarrow$  (i) and also (iii)  $\Rightarrow$  (iv)  $\Rightarrow$  (i).

If  $\alpha$  attacks  $\beta$ , a coherent imprecise probability satisfies conditions (ii) and (iv), hence also (i), as shown by Proposition 5.

**Proposition 5** *Let  $AF$  be an argumentation framework  $\langle \mathcal{A}, \rightarrow \rangle$ ,  $\underline{P}$  be a coherent lower probability defined on a set of events  $E \subseteq E_{ac}(\mathcal{A}) \cup E_{-ac}(\mathcal{A})$  and  $\overline{P}$  its conjugate upper probability. Whenever the relevant events belong to  $E$ , it holds that if  $\alpha \rightarrow \beta$  then  $\underline{P}(\hat{\alpha}) \leq \underline{P}(\neg\hat{\beta})$  and  $\overline{P}(\hat{\alpha}) \leq \overline{P}(\neg\hat{\beta})$ .*

**Proof:** From  $\alpha \rightarrow \beta$  it follows  $\hat{\alpha} \Rightarrow \neg\hat{\beta}$ . As mentioned at the end of Sec. 1, it is a well-known monotonicity property of coherent lower (and upper) probabilities ([10] Sec. 2.7.4 (c)) that if  $e_1 \Rightarrow e_2$  then  $\underline{P}(e_1) \leq \underline{P}(e_2)$  and  $\overline{P}(e_1) \leq \overline{P}(e_2)$ .  $\square$

Condition (iii) instead is not implied by coherence. For instance the vacuous lower probability is coherent but does not satisfy it:  $\overline{P}(\hat{\alpha}) = 1 - \underline{P}(\neg\hat{\alpha}) = 1 > 0 = \underline{P}(\neg\hat{\beta})$ .

Condition (ii) corresponds to the strongest interpretation of weak p-justifiability in presence of imprecision: it entails that any precise probability  $P$  compatible with the given imprecise assessment (i.e.  $\underline{P}(e) \leq P(e) \leq \overline{P}(e) \forall e$ ) is weakly p-justifiable. To see this, note that  $\underline{P}(\hat{\alpha}) \leq \overline{P}(\hat{\alpha}) \leq \underline{P}(\neg\hat{\beta}) \leq P(\neg\hat{\beta})$ , the first and third inequality holding by the assumed compatibility of  $P$ , the second by condition (iii). Then  $P(\hat{\alpha}) \leq P(\neg\hat{\beta})$ , which is the weak p-justifiability of  $P$ .

On the other hand conditions (i), (ii) and (iv) do not ensure that there is at least one precise probability compatible with the considered imprecise assessment but do not even prevent this possibility, and condition (i) is the least committed among them. Hence, it seems reasonable to consider conditions (i) and (iii) as, respectively, the partial and full counterpart of weak p-justifiability in the context of imprecise probability assessments. This motivates the following definition, where (iii) is reformulated using the conjugacy relation (3).

**Definition 10** *Let  $AF$  be an argumentation framework  $\langle \mathcal{A}, \rightarrow \rangle$ ,  $\underline{P}$  be a lower probability defined on a set of events  $E \subseteq E_{ac}(\mathcal{A}) \cup E_{-ac}(\mathcal{A})$ . Whenever the relevant events belong to  $E$ :*

$\underline{P}$  is partially wp-justifiable iff  $\forall \beta \in \{\alpha\}^- \underline{P}(\hat{\alpha}) \leq 1 - \underline{P}(\hat{\beta})$ ;

$\underline{P}$  is fully wp-justifiable iff  $\forall \beta \in \{\alpha\}^- 1 - \underline{P}(\neg\hat{\alpha}) \leq \underline{P}(\neg\hat{\beta})$ .

The following property follows directly from Proposition 5.

**Property 2** *Let  $AF$  be an argumentation framework  $\langle \mathcal{A}, \rightarrow \rangle$ . Every coherent lower probability defined on a set of events  $E \subseteq E_{ac}(\mathcal{A}) \cup E_{-ac}(\mathcal{A})$  is partially wp-justifiable.*

As already mentioned, full wp-justifiability is instead not implied by coherence. Given that coherent lower probabilities include coherent precise probabilities as a special

case, the existence of at least one fully wp-justifiable coherent lower probability for every  $AF$  follows from Proposition 3.

Turning to second constituent condition of p-justifiability, we consider its revised version as in equation (5):  $P(\hat{\alpha}) \geq 1 - P(\bigvee_{\beta \in \{\alpha\}^-} \hat{\beta})$  which in the precise case equivalently reads as  $P(\hat{\alpha}) \geq P(\bigwedge_{\beta \in \{\alpha\}^-} \neg \hat{\beta})$ . As above, the two formulations in the precise case give rise to four formulations in the imprecise case, namely:

- (i)  $\underline{P}(\hat{\alpha}) \geq 1 - \underline{P}(\bigvee_{\beta \in \{\alpha\}^-} \hat{\beta})$ ; (ii)  $\underline{P}(\hat{\alpha}) \geq \underline{P}(\bigwedge_{\beta \in \{\alpha\}^-} \neg \hat{\beta})$ ;
- (iii)  $\overline{P}(\hat{\alpha}) \geq 1 - \overline{P}(\bigvee_{\beta \in \{\alpha\}^-} \hat{\beta})$ ; (iv)  $\overline{P}(\hat{\alpha}) \geq \overline{P}(\bigwedge_{\beta \in \{\alpha\}^-} \neg \hat{\beta})$ .

Noting that, by (3), (i) is equivalent to  $\underline{P}(\hat{\alpha}) \geq \overline{P}(\bigwedge_{\beta \in \{\alpha\}^-} \neg \hat{\beta})$  and (iii) to  $\overline{P}(\hat{\alpha}) \geq \underline{P}(\bigwedge_{\beta \in \{\alpha\}^-} \neg \hat{\beta})$ , it is easy to see that (i)  $\Rightarrow$  (ii)  $\Rightarrow$  (iii) and (i)  $\Rightarrow$  (iv)  $\Rightarrow$  (iii). Applying considerations analogous to the previous case, one can then note that (i) ensures that all the precise probabilities compatible with  $\underline{P}$  satisfy equation (5), while (iii) is the weakest among the other conditions.

Accordingly, we can propose the notions of partial and full rp-justifiability.

**Definition 11** *Let  $AF$  be an argumentation framework  $\langle \mathcal{A}, \rightarrow \rangle$ ,  $\underline{P}$  be a lower probability defined on a set of events  $E \subseteq E_{ac}(\mathcal{A}) \cup E_{-ac}(\mathcal{A})$ . Whenever the relevant events belong to  $E$ :*

*$\underline{P}$  is partially rp-justifiable iff it is partially wp-justifiable and  $1 - \underline{P}(\neg \hat{\alpha}) \geq \underline{P}(\bigwedge_{\beta \in \{\alpha\}^-} \neg \hat{\beta})$ ;*

*$\underline{P}$  is fully rp-justifiable iff it is fully wp-justifiable and  $\underline{P}(\hat{\alpha}) \geq 1 - \underline{P}(\bigvee_{\beta \in \{\alpha\}^-} \hat{\beta})$ .*

Since  $\hat{\alpha} \Rightarrow \bigwedge_{\beta \in \{\alpha\}^-} \neg \hat{\beta}$ , in case  $\underline{P}$  is coherent (hence monotonic) and is fully rp-justifiable we get (using monotonicity for the last inequality)  $\overline{P}(\hat{\alpha}) \geq \underline{P}(\hat{\alpha}) \geq \overline{P}(\bigwedge_{\beta \in \{\alpha\}^-} \neg \hat{\beta}) \geq \overline{P}(\hat{\alpha})$  and similarly  $\underline{P}(\hat{\alpha}) \geq \overline{P}(\bigwedge_{\beta \in \{\alpha\}^-} \neg \hat{\beta}) \geq \underline{P}(\bigwedge_{\beta \in \{\alpha\}^-} \neg \hat{\beta}) \geq \underline{P}(\hat{\alpha})$ , from which  $\overline{P}(\hat{\alpha}) = \underline{P}(\hat{\alpha}) = \overline{P}(\bigwedge_{\beta \in \{\alpha\}^-} \neg \hat{\beta}) = \underline{P}(\bigwedge_{\beta \in \{\alpha\}^-} \neg \hat{\beta})$ . This suggests that, unless coherence is violated, full rp-justifiability turns out to be an overly restrictive property for imprecise probabilities, since it, actually, tends to force precise assessments on the relevant events.

Thus, only partial rp-justifiability appears to have potential interest in the case of imprecise assessments. While a detailed analysis of this property is left to future work, we observe that, differently from what noted in Section 2, partial rp-justifiability for imprecise probabilities does not fall into impossibility in the case of  $AF_3$ , confirming the well-known greater flexibility of imprecise probabilities in uncertainty representation. More specifically, it can be proved that partial rp-justifiability implies  $\overline{P}(\hat{\alpha}) = \overline{P}(\hat{\beta}) = \overline{P}(\hat{\gamma})$  in this case, while it allows (but does not enforce)  $\underline{P}(\hat{\alpha})$ ,  $\underline{P}(\hat{\beta})$ , and  $\underline{P}(\hat{\gamma})$  to differ. To exemplify this, using the same notation as in Section 2, consider three precise assessments  $P' = (p_1, p_2, p_3, p_4) = (0.6, 0.2, 0.2, 0)$ ,  $P'' = (0.2, 0.6, 0.2, 0)$ , and  $P''' = (0.1, 0.6, 0.2, 0.1)$  and their lower envelope  $\underline{P}$  (coherent in virtue of Theorem 2). It is easy to see that  $\underline{P}(\hat{\alpha}) = 0.1$ ,  $\underline{P}(\hat{\beta}) = \underline{P}(\hat{\gamma}) = 0.2$ ,  $\overline{P}(\hat{\alpha}) = \overline{P}(\hat{\beta}) = \overline{P}(\hat{\gamma}) = 0.6$  and that both the first and second condition for partial rp-justifiability are satisfied. A symmetry on  $\underline{P}$  too can be obtained by an obvious modification of  $P'''$ . Investigating stronger conditions ensuring symmetry on lower probabilities in this and in similar cases, while not running into the problems of full rp-justifiability, is left to future work.

## 4. Conclusions

We have carried out a preliminary analysis showing that the study of epistemic probabilities in abstract argumentation can be conveniently related to the theory of coherent subjective probabilities developed by de Finetti [2] and extended by Walley [10] to the case of imprecise assessments. In fact, coherent subjective probability theory provides an extensive theoretical corpus of definitions and results encompassing into a unitary conceptual framework either partial (on arbitrary sets of events) or complete (on entire algebraic structures of events) precise and imprecise probability assessments, and thus ensures the coverage of a large variety of application contexts.

As a starting point for this research line, we have focused attention on the rationality conditions for epistemic probabilities proposed in [9] and shown how their initial definition can be reassessed, revised, and extended to the case of lower probabilities. After relating the two technical justifiability conditions with the properties of conflict-freeness and reinstatement, we have seen in particular that the property of coherence implies a weak notion of justifiability for epistemic (precise and imprecise) probabilities, while the full notion of justifiability in [9] appears rather strong and may require some revision. Moreover, we have shown that, going from precise to imprecise probabilities, alternative formulations of the notion of justifiability can be considered. Future work directions include a more extensive analysis of the variety of rationality conditions in the case of imprecise probabilities and the investigation of the relations between (possibly stronger) rationality conditions and other traditional semantics properties besides conflict-freeness.

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