Preferences and Unrestricted Rebut

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Abstract. The work of Caminada & Amgoud presents two possible ways of satisfying the *rationality postulates*: one using *restricted rebut*, and one using *unrestricted rebut*. Subsequent work on ASPIC+ has extended the work of Caminada & Amgoud, for instance by allowing preferences over arguments. However, such extensions have utilised restricted rebut only. In the current paper, we make the case for unrestricted rebut, and provide a formalism (called ASPIC-) that implements preferences between the defeasible rules, in the context of unrestricted rebut while still satisfying the rationality postulates of Caminada & Amgoud.

1. Introduction

One of the important aspects of instantiated argumentation [2,9,10,12] is how to precisely determine whether one argument attacks another. Suppose one considers arguments constructed using *reasons*, represented as rules. Many rules will be defeasible (for instance, when applying the argument scheme of expert opinion), but some rules will be strict (for instance, *Modus ponens*).The idea in formalisms like ASPIC+ [12,15], ASPIC^{lite} [19] and ABA [9] is that arguments are constructed by chaining rules, so that the consequent of one rule feeds into the antecedent of the next. In this way, an argument is essentially a derivation supporting a particular conclusion (namely the consequent of the top rule).

Given this method of argument construction, the next step is to define the attacks between arguments. For this, ASPIC-like formalisms distinguish between undercut-attacks (where the attacking argument derives the non-applicability of a particular defeasible rule in the attacked argument) and rebut-attacks (where the attacking argument derives the contrary (negation) of one of the (intermediate) conclusions of the attacked argument.

In the literature, one can observe two principles for defining rebut-attacks: *restricted rebut* and *unrestricted rebut* [2]. To illustrate the difference between these forms of rebut, consider the example of arguments $A = (\Rightarrow a) \rightarrow b$ and $B = (\rightarrow c) \Rightarrow \neg b$.[1] With restricted rebut, one can only rebut-attack a conclusion that is the consequent of a defeasible rule. Hence, A attacks (restrictedly rebuts) B, but B does not attack (restrictedly rebut) A. However, with unrestricted rebut, one can rebut-attack *any* conclusion (even consequents of strict rules) as long as at least one defeasible rule has been used in its derivation. So A still attacks (unrestrictedly rebuts) B, but B also attacks (unrestrictedly rebuts) A. In essence, with restricted rebut, the *last* rule of the attacked conclusion must be defeasible, whereas with unrestricted rebut one merely requires that *any previous rule*

[1] We are applying ASPIC+ notation, with argument A consisting of a defeasible rule with empty antecedent "$\Rightarrow a$", followed by a strict rule $a \rightarrow b$, and argument B consisting of a strict rule with empty antecedent "$\rightarrow c$", followed by a defeasible rule "$c \Rightarrow \neg b$".

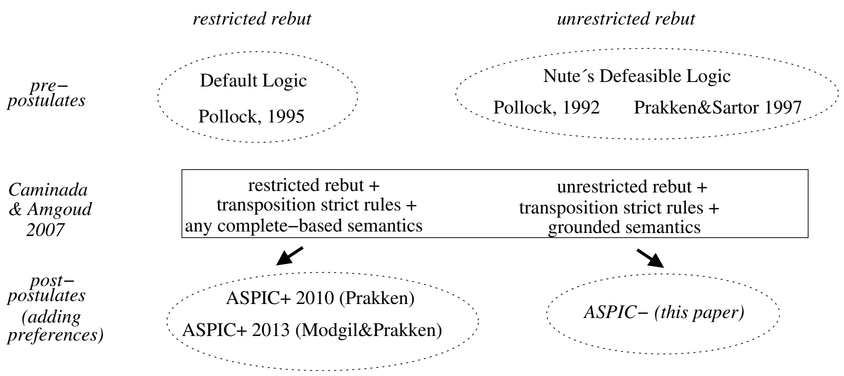

Figure 1. Research lines on rule-based argumentation.

(possibly even the last one) is defeasible. Hence, if an argument restrictedly rebuts another argument then it also unrestrictedly rebuts it, but not vice versa. Forms of unrestricted rebut are applied in the formalism of Prakken and Sartor [16], the argumentation version of Nute's Defeasible Logic [11,13] and [14]'s version of Pollock's OSCAR system. Forms of restricted rebut are applied in the 1995 version of Pollock's OSCAR system, as well as in the argumentation version of Default Logic [3,8,17] and ASPIC+[12].

When it comes to reasoning with strict and defeasible rules, one of the landmark papers is the work of Caminada and Amgoud [2], in which a number of *rationality postulates* are specified: *direct consistency*, *indirect consistency*, *(conclusion) closure* and *subargument closure*. The authors provide two different approaches for satisfying these postulates: one that applies unrestricted rebut, and one that applies restricted rebut. The approach that applies unrestricted rebut is shown to satisfy the postulates only under grounded semantics. The approach that applies restricted rebut is shown to satisfy the postulates under any complete-based semantics[2]. This paper focuses on how to define *unrestricted* rebut-attacks in a slightly simplified version of [12]'s ASPIC+ framework[3] under grounded semantics.

In constructing ASPIC+, Prakken and Modgil build on the work of [2] by allowing preferences over arguments, and proving that the rationality postulates of [2] are still satisfied [12,15] under certain conditions on the preference relation. They then show that various ways of defining preferences based on a prioritisation of defeasible knowledge, satisfy these conditions. However, the postulates are shown to hold only when applying restricted rebut. This leaves open the question of how to allow preferences when applying unrestricted rebut. It is this question that is studied in the current paper. An overview of how the current work fits into the existing research lines is provided in figure 1.

The remainder of this paper is structured as follows. In Section 2, we provide the formal preliminaries of a slightly simplified version of ASPIC+ (called ASPIC-) that applies unrestricted rebut and allows for preferences over arguments as defined in [12]. Section 3 then discusses why the approach of restricted rebut (as applied in ASPIC+) is not satisfactory, and makes the case for unrestricted rebut. In Section 4, we show that

[2]What we mean is any semantics whose extensions are complete. Examples of such are complete, grounded, preferred, stable, semi-stable, ideal and eager semantics, but not stage and CF2 semantics.

[3]The choice of ASPIC+ is motivated by the level of generality that the framework affords, in that a number of existing approaches to argumentation have been shown to be instances of the framework [12,15]

under the approach of unrestricted rebut (as applied in our ASPIC- formalism), the rationality postulates are satisfied under the assumption that priority orderings over defeasible knowledge are total orderings. In Section 5, we conclude with a discussion.

2. Preliminaries

In what follows we define ASPIC- as a special case of [12]'s ASPIC+ framework. Specifically, arguments in [12] are constructed from defeasible and strict rules, *and* ordinary and axiom premises, of which only the the former can be attacked. ASPIC- models ordinary premises as antecedent free defeasible inference rules, and axiom premises as antecedent free strict inference rules. Furthermore, we assume a language closed under negation ($\neg$) rather than [12]'s more general contrary 'function' which identifies when two formulae in the given language are in conflict. Other than these simplifications, the other key difference is that unlike [12], ASPIC- allows *unrestricted rebuts*.

Definition 1. *An* argumentation system *is a tuple* $AS = (\mathcal{L}, \mathcal{R}, \mathtt{n})$ *where:*

- $\mathcal{L}$ *is a logical language closed under negation ($\neg$).*
- $\mathcal{R} = \mathcal{R}_s \cup \mathcal{R}_d$ *is a finite set of strict ($\mathcal{R}_s$) and defeasible ($\mathcal{R}_d$) inference rules of the form* $\varphi_1, \ldots, \varphi_n \rightarrow \varphi$ *and* $\varphi_1, \ldots, \varphi_n \Rightarrow \varphi$ *respectively (where* φ_i, φ *are meta-variables ranging over wff in* $\mathcal{L}$*), and* $\mathcal{R}_s \cap \mathcal{R}_d = \emptyset$*.* $\mathcal{R}_s$ *is assumed to be closed under transposition, i.e., if* $\phi_1, \ldots, \phi_n \rightarrow \psi \in \mathcal{R}_s$*, then for* $i = 1 \ldots n$*,* $\phi_1, \ldots, \phi_{i-1}, -\psi, \phi_{i+1}, \ldots, \phi_n \rightarrow -\phi_i \in \mathcal{R}_s$*.*
- $\mathtt{n}$ *is a partial function such that* $\mathtt{n} : \mathcal{R}_d \longrightarrow \mathcal{L}$*.*

We write $\psi = -\varphi$ *just in case* $\psi = \neg\varphi$ *or* $\varphi = \neg\psi$ *(we will sometimes informally say that formulas* φ *and* $-\varphi$ *are each other's negation).*

Definition 2. *An* argument A *on the basis of an argumentation system* $(\mathcal{L}, \mathcal{R}, \mathtt{n})$ *is defined as:*

1. $A_1, \ldots A_n \rightarrow \psi$ *if* $A_1, \ldots, A_n$ *(*$n \geq 0$*) are arguments, and there is a strict rule* $\mathtt{Conc}(A_1), \ldots, \mathtt{Conc}(A_n) \rightarrow \psi$ *in* $\mathcal{R}_s$*. In that case, we define*
   $\mathtt{Conc}(A) = \psi$,
   $\mathtt{Sub}(A) = \mathtt{Sub}(A_1) \cup \ldots \cup \mathtt{Sub}(A_n) \cup \{A\}$.
   $\mathtt{DefRules}(A) = \mathtt{DefRules}(A_1) \cup \ldots \cup \mathtt{DefRules}(A_n)$,
   $\mathtt{TopRule}(A) = \mathtt{Conc}(A_1), \ldots \mathtt{Conc}(A_n) \rightarrow \psi$
2. $A_1, \ldots A_n \Rightarrow \psi$ *if* $A_1, \ldots, A_n$ *(*$n \geq 0$*) are arguments, and there exists a defeasible rule*
   $\mathtt{Conc}(A_1), \ldots, \mathtt{Conc}(A_n) \Rightarrow \psi$ *in* $\mathcal{R}_d$*. In that case, we define*
   $\mathtt{Conc}(A) = \psi$,
   $\mathtt{Sub}(A) = \mathtt{Sub}(A_1) \cup \ldots \cup \mathtt{Sub}(A_n) \cup \{A\}$,
   $\mathtt{DefRules}(A) = \mathtt{DefRules}(A_1) \cup \ldots \cup \mathtt{DefRules}(A_n) \cup \{\mathtt{Conc}(A_1), \ldots \mathtt{Conc}(A_n) \Rightarrow \psi\}$,
   $\mathtt{TopRule}(A) = \mathtt{Conc}(A_1), \ldots \mathtt{Conc}(A_n) \Rightarrow \psi$.

Furthermore, for any argument A *and set of arguments* E*:*

- A *is* strict *iff* $\mathtt{DefRules}(A) = \emptyset$*;* defeasible *iff* $\mathtt{DefRules}(A) \neq \emptyset$*;*
- *If* $\mathtt{DefRules}(A) = \emptyset$*, then* $\mathtt{LastDefRules}(A) = \emptyset$*, else;*
  if $A = A_1, \ldots, A_n \Rightarrow \phi$ *then* $\mathtt{LastDefRules}(A) = \{\mathtt{Conc}(A_1), \ldots, \mathtt{Conc}(A_n) \Rightarrow \phi\}$*, otherwise* $\mathtt{LastDefRules}(A) = \mathtt{LastDefRules}(A_1) \cup \ldots \cup \mathtt{LastDefRules}(A_n)$.

- *if* $\texttt{TopRule}(A) = \texttt{Conc}(A_1), \ldots \texttt{Conc}(A_n) \rightarrow \psi$*, then* A *is said to be a* strict extension *of* $\{A_1, \ldots, A_n\}$.
- $\texttt{Concs}(E) = \{\texttt{Conc}(A) | A \in E\}$
- *The closure under strict rules of* E*, denoted* $Cl_S(E)$ *is the smallest set containing* $\texttt{Concs}(E)$ *and the consequent of any strict rule in* $\mathcal{R}_s$ *whose antecedent is contained in* $Cl_S(E)$.

As in [2,15] we henceforth assume that the strict knowledge in an argumentation system is consistent in the following sense.

Definition 3. *Let* Ar' *be the set of all* strict *arguments defined on the basis of* $AS = (\mathcal{L}, \mathcal{R}, \texttt{n})$. *Then* AS *is said to be consistent iff* $\neg\exists A, B \in Ar'$ *such that* $\texttt{Conc}(A) = -\texttt{Conc}(B)$

In the following, attacks on the conclusions of rules (*rebuts*) are not restricted to the consequents of defeasible rules, and so are referred to as *unrestricted*.

Definition 4. A attacks B *iff* A undercuts *or* rebuts B*, where:*

- A undercuts *argument* B *(on* B'*) iff* $\texttt{Conc}(A) = -\texttt{n}(r)$ *for some* $B' \in \texttt{Sub}(B)$ *such that* $\texttt{TopRule}(B') = r$ *where* r *is defeasible.*
- A rebuts *argument* B *(on* B'*) iff* $\texttt{Conc}(A) = -\texttt{Conc}(B')$ *for some* $B' \in \texttt{Sub}(B)$[4]

Given a preference ordering over arguments, a rebut attack succeeds as a *defeat* (see Definition 7) only if the attacked argument is not strictly preferred to its attacker. Undercut attacks always succeed as defeats, irrespective of preferences (see [12] for further details). We now recapitulate examples of how [12] define a preference ordering $\preceq$ over arguments, based on a priority ordering over the arguments' defeasible rules. However, unlike [12], we assume a *total* pre-ordering on the defeasible rules. This ordering is used to compare sets of all the defeasible rules in the compared arguments (the *weakest link* principle) or only the top most defeasible rules (the *last link* principle). These rule set orderings are of two types; the so called `Elitist` and `Democratic` set orderings.

Definition 5. *Let* $\leq \subseteq (\mathcal{R}_d \times \mathcal{R}_d)$ *be a total pre-ordering on the defeasible inference rules, where as usual,* $r < r'$ *iff* $r \leq r'$ *and* $r \not\leq r'$*, and* $r \equiv r'$ *iff* $r \leq r'$ *and* $r' \leq r$. *Then for any* $\mathcal{E}, \mathcal{E}' \subseteq \mathcal{R}_d$, $\trianglelefteq_{\texttt{s}}$ *(*$\texttt{s} \in \{\texttt{Eli}, \texttt{Dem}\}$*) is defined as follows:*

1. *If* $\mathcal{E} = \emptyset$ *then* $\mathcal{E} \ntrianglelefteq_{\texttt{s}} \mathcal{E}'$ *;*
2. *If* $\mathcal{E}' = \emptyset$ *and* $\mathcal{E} \neq \emptyset$ *then* $\mathcal{E} \trianglelefteq_{\texttt{s}} \mathcal{E}'$ *; else:*
3. *if* $\texttt{s} = \texttt{Eli}$*:* $\mathcal{E} \trianglelefteq_{\texttt{Eli}} \mathcal{E}'$ *if* $\exists A \in \mathcal{E}$ *s.t.* $\forall B \in \mathcal{E}'$, $A \leq B$. *else:*
4. *if* $\texttt{s} = \texttt{Dem}$*:* $\mathcal{E} \trianglelefteq_{\texttt{Dem}} \mathcal{E}'$ *if* $\forall A \in \mathcal{E}$, $\exists B \in \mathcal{E}'$, $A \leq B$.

As usual $\mathcal{E} \triangleleft_{\texttt{s}} \mathcal{E}'$ *iff* $\mathcal{E} \trianglelefteq_{\texttt{s}} \mathcal{E}'$ *and* $\mathcal{E}' \ntrianglelefteq_{\texttt{s}} \mathcal{E}$

The democratic ordering has been used in Dung-style argumentation since the work of [16]. The elitist ordering was introduced into formal argumentation more recently, and has for instance been shown in [12] to support argument-based interpretations of Brewka's preferred subtheories [1]. Both orderings have been used within ASPIC+.

[4]Note that whereas the principle of unrestricted rebut commonly requires the attacked argument to be defeasible, no such requirement has explicitly been stated in Definition 4. However, as in [12], no rebut-attack on a strict argument will succeed (and constitute a *defeat*) since all the preference principles examined in the remainder of this paper are such that strict arguments are maximally preferred (see Proposition 1).

Definition 6. *Let Ar be defined on the basis of $(\mathcal{L}, \mathcal{R}, \mathtt{n})$. Then $\forall A, B \in Ar$:*

1. $A \preceq_{\mathtt{Ewl}} B$ *iff* $\mathtt{DefRules}(A) \trianglelefteq_{\mathtt{Eli}} \mathtt{DefRules}(B)$
2. $A \preceq_{\mathtt{Ell}} B$ *iff* $\mathtt{LastDefRules}(A) \trianglelefteq_{\mathtt{Eli}} \mathtt{LastDefRules}(B)$
3. $A \preceq_{\mathtt{Dwl}} B$ *iff* $\mathtt{DefRules}(A) \trianglelefteq_{\mathtt{Dem}} \mathtt{DefRules}(B)$
4. $A \preceq_{\mathtt{Dll}} B$ *iff* $\mathtt{LastDefRules}(A) \trianglelefteq_{\mathtt{Dem}} \mathtt{LastDefRules}(B)$

where Ewl, Ell, Dwl *and* Dll *respectively denote '*Elitist *weakest link', '*Elitist *last link', '*Democratic *weakest link' and '*Democratic *last link'.*

We may write $A \prec_p B$ iff $A \preceq_p B$ and $B \npreceq_p A$; and write $A \approx_p B$ iff $A \preceq_p B, B \preceq_p A$ (where $p \in \{\mathtt{Ewl}, \mathtt{Ell}, \mathtt{Dwl}, \mathtt{Dll}\}$). It is straightforward to show that $\prec_p$ is a strict partial ordering (irreflexive, transitive and asymmetric).

Proposition 1. *Let Ar be defined on the basis of $(\mathcal{L}, \mathcal{R}, n)$. Then $\forall A, B \in Ar$:*

1. *If B is strict and A is defeasible, then $A \prec_p B$.*
2. *If B is strict then $B \nprec_p A$.*

Proof. 1): By assumption, $\mathcal{E}$ = $(\mathtt{Last})\mathtt{DefRules}(A) \neq \emptyset$, and $\mathcal{E}'$ = $(\mathtt{Last})\mathtt{DefRules}(B)$ = $\emptyset$. By Definition 5-1 & 5-2, $\mathcal{E}' \ntrianglelefteq_{\mathtt{s}} \mathcal{E}$ and $\mathcal{E} \trianglelefteq_{\mathtt{s}} \mathcal{E}'$. By Definition 6, $A \preceq_p B$ and $B \npreceq_p A$, and so $A \prec_p B$.
2): Let $\mathcal{E}$ = $(\mathtt{Last})\mathtt{DefRules}(B)$ = $\emptyset$. By Definition 5-1, $\forall \mathcal{E}'$, $\mathcal{E} \ntriangleleft_{\mathtt{s}} \mathcal{E}'$. Hence by Definition 6, $\forall A$, $B \nprec_p A$. □

Definition 7. *Let Ar be defined on the basis of $(\mathcal{L}, \mathcal{R}, \mathtt{n})$. Let $\leq$ be a total pre-ordering on $\mathcal{R}_d$ and let $\preceq_p$ be the associated preference ordering over Ar, as defined in Definition 6. Then $def \subseteq Ar \times Ar$ is said to be defined over Ar, where:*
$(A, B) \in def$ iff either A undercuts B, or A rebuts B and $A \nprec_p B$.
We say that the Dung framework (Ar, def) is defined by $(\mathcal{L}, \mathcal{R}, \mathtt{n})$ and $\leq$.

Definition 8. *Let (Ar, def) be defined by $(\mathcal{L}, \mathcal{R}, \mathtt{n})$ and $\leq$. Let $A \in Ar$ and $E \subseteq Ar$. E is said to be* conflict-free *iff there do not exist a $B, C \in E$ such that $(B, C) \in def$. E is said to* defend A *iff for every $B \in Ar$ such that $(B, A) \in def$ there exists an $C \in E$ such that $(C, B) \in def$. The characteristic function $F : 2^{Ar} \rightarrow 2^{Ar}$ is defined as $F(E) = \{A \in Ar \mid E \text{ defends } A\}$. E is called:*

- *an* admissible *set iff E is conflict-free and $E \subseteq F(E)$*
- *a* complete extension *iff E is conflict-free and $E = F(E)$*
- *a* grounded extension *iff E is the minimal (w.r.t. set-inclusion) complete extension*
- *a* preferred extension *iff E is a maximal (w.r.t. set-inclusion) complete extension;*

3. A Criticism of the Principle of Restricted Rebut

The following discussion highlights the advantages of unrestricted over restricted rebut.

John: *"Bob will attend conferences A and I this year, as he has papers accepted at both."*
Mary: *"That won't be possible, as his budget of £1000 only allows for one foreign trip."*

Formally, this discussion could be modelled using the argumentation theory $(\mathcal{L}, \mathcal{R}, \mathtt{n})$ with $\mathcal{R}_d \supseteq \{\text{accA} \Rightarrow \text{attA};\ \text{accI} \Rightarrow \text{attI};\ \text{budget} \Rightarrow \neg(\text{attA} \wedge \text{attI})\}$ and $\mathcal{R}_s \supseteq \{\rightarrow \text{accA};\ \rightarrow \text{accI};\ \rightarrow \text{budget};\ \text{attA}, \text{attI} \rightarrow \text{attA} \wedge \text{attI}\}$.

John: $((\to \text{accA}) \Rightarrow \text{attA}), ((\to \text{accI}) \Rightarrow \text{attI}) \to \text{attA} \wedge \text{attI}$
Mary: $(\to \text{budget}) \Rightarrow \neg(\text{attA} \wedge \text{attI})$
In ASPIC-, Mary's argument attacks John's argument, as one would expect in natural discussion. Such an attack utilises *unrestricted rebut*. That is, in ASPIC+ or any other formalism based on restricted rebut, Mary's argument does *not* attack John's argument, since the conclusion Mary wants to attack ($\text{attA} \wedge \text{attI}$) is the consequent of a strict rule. Under ASPIC+, if Mary wants to attack John's argument, she can only do so by attacking the consequent of a defeasible rule. That is, she would be forced to choose to attack either attA or attI, meaning she essentially has to utter one of the following two statements.

Mary$'$: *"Bob will attend I, so can't attend A; his budget doesn't allow him to attend both."*
Mary$''$: *"Bob will attend A, so can't attend I; his budget doesn't allow him to attend both."*
The associated formal counter arguments are as follows.[5]

Mary$'$: $((\to \text{budget}) \Rightarrow \neg(\text{attA} \wedge \text{attI})), ((\to \text{accI}) \Rightarrow \text{attI}) \to \neg\text{attA}$
Mary$''$: $((\to \text{accA}) \Rightarrow \text{attA}), ((\to \text{budget}) \Rightarrow \neg(\text{attA} \wedge \text{attI})) \to \neg\text{attI}$
Critically, Mary does not *know* which of the two conferences Bob will attend, yet the principle of restricted rebut *forces* her to make concrete statements on this. From the perspective of commitment in dialogue [18], this is unnatural. One should not be forced to commit to things one has insufficient reasons to believe in.

It should be stressed that the problem outlined above is particularly relevant in dialectical contexts, where different agents make commitments during the exchange of arguments. This contrasts with a formalism like ASPIC+, which is more monolithic in nature, in that from the given rules and premises, one constructs a Dung graph of attacking arguments and simply *computes* which arguments (and associated conclusions) are justified. Concepts like different agents, communication steps or commitment stores do not play a role in ASPIC+, and restricted rebut therefore *seems* acceptable. However, if one wants to add dialectical aspects to formal argumentation (c.f., [6,5,4]) then one is forced to take the limitations of restricted rebut seriously.

4. Postulates

We now prove that under unrestricted rebut, ASPIC- satisfies [2]'s rationality postulates when applying grounded semantics, assuming the preference orderings defined above.

Theorem 1. *Let* (Ar, def) *be defined by a consistent* $(\mathcal{L}, \mathcal{R}, \mathtt{n})$ *and* $\leq$*, and let* E *be the grounded extension of* (Ar, def). $\mathtt{Concs}(E)$ *satisfies* direct consistency*:* $\neg\exists\alpha$ *such that* $\alpha, \neg\alpha \in \mathtt{Concs}(E)$.

Proof. Suppose $\alpha, \neg\alpha \in \mathtt{Concs}(E)$. Then there exist two arguments $A, B \in E$ with $\mathtt{Conc}(A) = \alpha$ and $\mathtt{Conc}(B) = \neg\alpha$. We distinguish four cases:

1. both A and B are strict. This cannot be the case as this would contradict the assumption that the strict knowledge $\mathcal{R}_s$ is consistent.
2. A is strict and B is defeasible. Then A and B rebut each other. By Proposition 1 (point 1), $A \not\prec_p B$ and so $(A, B) \in \mathit{def}$, contradicting that E is conflict free.
3. A is defeasible and B is strict. This case is similar to the second one.

[5] Since $\mathcal{R}_s$ is assumed to be closed under transposition, the fact that $\mathcal{R}_s$ contains $\text{attA}, \text{attI} \to \text{attA} \wedge \text{attI}$ implies that $\mathcal{R}_s$ also contains $\neg(\text{attA} \wedge \text{attI}), \text{attI} \to \neg\text{attA}$ and $\text{attA}, \neg(\text{attA} \wedge \text{attI}) \to \neg\text{attI}$.

4. Both A and B are defeasible. The fact that $\prec_p$ is asymmetric implies that $A \not\prec_p B$ or $B \not\prec_p A$. Assume without loss of generality that $A \not\prec_p B$. Then A defeats B, contradicting that E is conflict free. □

The next property to be proved is that of *subargument closure*.

Theorem 2. *Let E be the grounded extension of (Ar, def). E is closed under subarguments. That is, if $A \in E$, then $\forall A' \in \texttt{Sub}(A)$, $A' \in E$.*

Proof. Let $A \in E$ and $A' \in \texttt{Sub}(A)$. Since E is admissible and $A \in E$, then every defeater B of A is defeated by an argument in E. Now, let B be an arbitrary defeater of A'. We distinguish two cases:

1. B undercuts A'. Then B also undercuts (and so defeats) A.
2. B rebuts A' and $B \not\prec_p A''$ (where A'' is the subargument of A' whose top-conclusion is rebutted by B). But this means that B also rebuts and defeats A.

So, every defeater of A' is defeated by an argument in E. This means that A' is acceptable w.r.t. E. Since E is a complete extension, this implies that $A' \in E$. □

To prove the postulate of *closure under strict rules*, we first define the G-function, similar to the characteristic function F (Definition 8). While $F(E)$ yields the arguments defended by E, $G(E)$ yields the arguments *strongly* defended by E. We show that for constructing the grounded extension either G or F can be applied. We then prove some properties of the set comparisons and preference relations defined in the previous section.

Definition 9. *Let (Ar, def) be an argumentation framework. We say that $A \in Ar$* strongly defeats *$B \in Ar$ iff A defeats B, and B does not defeat A. We say that $Args \subseteq Ar$* strongly defends *$A \in Ar$ iff each $B \in Ar$ that defeats A is strongly defeated by some $C \in Args$. Let $G : 2^{Ar} \to 2^{Ar}$ be the function such that $G(Args) = \{A \in Ar \mid A$ is strongly defended by $Args\}$.*

Theorem 3. *Let F^i be an abbreviation of $F^i(\emptyset)$ and let G^i be an abbreviation of $G^i(\emptyset)$. It holds that for every $i \geq 0$: $F^i = G^i$.*

Proof. We prove this by induction over i. Since we need to go two levels down in the induction step, we need two basis steps.

Basis 0 Let $i = 0$. It holds that $F^0 = \emptyset$ and $G^0 = \emptyset$ so $F^0 = G^0$.

Basis 1 Let $i = 1$. It holds that $F^1 = \{A \in Ar \mid \neg\exists B \in Ar : B\ def\ A\}$ and $G^1 = \{A \in Ar \mid \neg\exists B \in Ar : B\ def\ A\}$, so $F^1 = G^1$.

Step Let $i \geq 1$. Suppose that both $F^i = G^i$ and $F^{i-1} = G^{i-1}$. We now prove that $F^{i+1} = G^{i+1}$.

$G^{i+1} \subseteq F^{i+1}$: Let $A \in G^{i+1}$. Then A is strongly defended by G^i. Therefore A is also (normally) defended by G^i. And since (induction hypothesis) $G^i = F^i$, it follows that A is also (normally) defended by F^i. Therefore, $A \in F^{i+1}$.

$F^{i+1} \subseteq G^{i+1}$: Let $A \in F^{i+1}$. Then A is (normally) defended by F^i. Suppose, towards a contradiction, that A is not strongly defended by F^i. Then there exists a defeater B of A that is not strongly defeated by F^i. This means that every $C \in F^i$ that defeats B is also defeated by B. The fact that A is (normally) defended by F^i implies that at least one such C exists. That is, there exists a $C \in F^i$ (and therefore a $C \in G^i$, since our induction hypothesis is that $F^i = G^i$)) that is defeated by B

and G^i does not strongly defeat this B. From the fact that $G^i \subseteq G^{i+1}$ it follows that G^{i-1} also does not strongly defeat B. But then G^{i-1} does not strongly defend C. Contradiction.

□

From the above theorem, it follows that $\cup_{i=0}^{\infty} F^i = \cup_{i=0}^{\infty} G^i$. This means we are free to compute the grounded extension using the G-function instead of the F-function.

In the following proofs (needed to prove Theorem 4 — *closure under strict rules*), lower case letters $a, b, c, \ldots$ denote defeasible rules. $\mathtt{(L)DR}(A)$ denote s$\mathtt{(Last)DefRules}(A)$, where the latter should be read as 'either $\mathtt{DefRules}(A)$ or $\mathtt{LastDefRules}(A)$'. For example, $\mathtt{(L)DR}(A) \trianglelefteq_p \mathtt{(L)DR}(B)$ should be read as '$\mathtt{DefRules}(A) \trianglelefteq_p \mathtt{DefRules}(B)$ or $\mathtt{LastDefRules}(A) \trianglelefteq_p \mathtt{LastDefRules}(B)$' respectively.

Lemma 1. *Let $\leq$ be a total pre-ordering on $\mathcal{R}_d$. Then $\forall \mathcal{E}, \mathcal{E}' \subseteq \mathcal{R}_d, \mathcal{E} \neq \emptyset, \mathcal{E}' \neq \emptyset$:*
1. $\mathcal{E} \triangleleft_{\mathtt{Eli}} \mathcal{E}'$ *iff* $\exists a \in \mathcal{E}, \forall b \in E', a < b$.
2. $\mathcal{E} \triangleleft_{\mathtt{Dem}} \mathcal{E}'$ *iff* $\forall a \in \mathcal{E}, \exists b \in E', a < b$.

Proof. Proof of 1: For the right to left half, given $\exists a \in \mathcal{E}$, $\forall b \in E'$, $a < b$, it follows that: i) $\exists a \in \mathcal{E}$, $\forall b \in E'$, $a \leq b$, hence $\mathcal{E} \trianglelefteq_{\mathtt{Eli}} \mathcal{E}'$; ii) it cannot be that $\exists b \in \mathcal{E}'$, $\forall a \in E$, $b \leq a$, hence $\mathcal{E}' \ntrianglelefteq_{\mathtt{Eli}} \mathcal{E}$. Hence $\mathcal{E} \triangleleft_{\mathtt{Eli}} \mathcal{E}'$.
For the left to right half, given $\mathcal{E} \triangleleft_{\mathtt{Eli}} \mathcal{E}'$, then $\mathcal{E} \trianglelefteq_{\mathtt{Eli}} \mathcal{E}'$ and $\mathcal{E}' \ntrianglelefteq_{\mathtt{Eli}} \mathcal{E}$. Let a be a $<$ minimal element in $\mathcal{E}$, i.e. $\neg \exists a'$ s.t. $a' < a$. By assumption of $\mathcal{E} \trianglelefteq_{\mathtt{Eli}} \mathcal{E}'$, and $\leq$ being a total ordering, $\forall b \in \mathcal{E}'$, $a \leq b$. Now suppose some $b \in \mathcal{E}'$ s.t. $a \equiv b$. Then $\forall a' \in \mathcal{E}$, $b \leq a'$, and so $\mathcal{E}' \trianglelefteq_{\mathtt{Eli}} \mathcal{E}$, contradicting $\mathcal{E}' \ntrianglelefteq_{\mathtt{Eli}} \mathcal{E}$. Hence, $\forall b \in \mathcal{E}'$, $a < b$.

Proof of 2: For the right to left half, given $\forall a \in \mathcal{E}$, $\exists b \in \mathcal{E}'$, $a < b$, it follows that: i) $\forall a \in \mathcal{E}$, $\exists b \in E'$, $a \leq b$, hence $\mathcal{E} \trianglelefteq_{\mathtt{Dem}} \mathcal{E}'$; ii) it cannot be that $\forall b \in \mathcal{E}'$, $\exists a \in E$, $b \leq a$, hence $\mathcal{E}' \ntrianglelefteq_{\mathtt{Dem}} \mathcal{E}$. Hence $\mathcal{E} \triangleleft_{\mathtt{Dem}} \mathcal{E}'$.
For the left to right half, given $\mathcal{E} \triangleleft_{\mathtt{Dem}} \mathcal{E}'$, then $\mathcal{E} \trianglelefteq_{\mathtt{Dem}} \mathcal{E}'$ and $\mathcal{E}' \ntrianglelefteq_{\mathtt{Dem}} \mathcal{E}$. Let b be a $<$ maximal element in $\mathcal{E}'$, i.e. $\neg \exists b'$ s.t. $b < b'$. By assumption of $\mathcal{E} \trianglelefteq_{\mathtt{Dem}} \mathcal{E}'$, and $\leq$ being a total ordering, $\forall a \in \mathcal{E}$, $a \leq b$. Now suppose some $a \in \mathcal{E}$ s.t. $a \equiv b$. Then $\forall b' \in \mathcal{E}'$, $b' \leq a$, and so $\mathcal{E}' \trianglelefteq_{\mathtt{Dem}} \mathcal{E}$, contradicting $\mathcal{E}' \ntrianglelefteq_{\mathtt{Dem}} \mathcal{E}$. Hence, $\forall a \in \mathcal{E}$, $a < b$. □

Lemma 2. $B \nprec_p A$ *iff* $\mathtt{(L)DR}(B) \ntriangleleft_{\mathtt{s}} \mathtt{(L)DR}(A)$.

Proof. Suppose $B \nprec_p A$ and $\mathtt{(L)DR}(B) \triangleleft_{\mathtt{s}} \mathtt{(L)DR}(A)$. Then $\mathtt{(L)DR}(B) \trianglelefteq_{\mathtt{s}} \mathtt{(L)DR}(A)$, $\mathtt{(L)DR}(A) \ntrianglelefteq_{\mathtt{s}} \mathtt{(L)DR}(B)$, and so $B \preceq_p A$, $A \npreceq_p B$, i.e., $B \prec_p A$. Contradiction.
Suppose $\mathtt{(L)DR}(B) \ntriangleleft_{\mathtt{s}} \mathtt{(L)DR}(A)$ and $B \prec_p A$. Then $B \preceq_p A$, $A \npreceq_p B$, and so $\mathtt{(L)DR}(B) \trianglelefteq_{\mathtt{s}} \mathtt{(L)DR}(A)$, $\mathtt{(L)DR}(A) \ntrianglelefteq_{\mathtt{s}} \mathtt{(L)DR}(B)$, i.e., $\mathtt{(L)DR}(B) \triangleleft_{\mathtt{s}} \mathtt{(L)DR}(A)$. Contradiction. □

Lemma 3. *Let A and B be arguments such that A is defeasible, B is strict or defeasible, and $B \nprec_p A$ where $p \in \{\mathtt{Ewl}, \mathtt{Ell}\}$. Then $\forall b \in \mathtt{(L)DR}(B), \exists a \in \mathtt{(L)DR}(A)$ s.t. $a \leq b$.*

Proof. If B is strict, the lemma is satisfied trivially.
Suppose B is defeasible. By Lemma 2, $\mathtt{(L)DR}(B) \ntriangleleft_{\mathtt{Eli}} \mathtt{(L)DR}(A)$. By Lemma 1, $\neg(\exists b \in \mathtt{(L)DR}(B)$ s.t. $\forall a \in \mathtt{(L)DR}(A), b < a)$. That is: $\forall b \in \mathtt{(L)DR}(B)$, $\exists a \in \mathtt{(L)DR}(A)$ s.t. $b \nless a$. Since $\leq$ is a total order: $\forall b \in \mathtt{(L)DR}(B), \exists a \in \mathtt{(L)DR}(A)$ s.t. $a \leq b$. □

Lemma 4. *Let A and B be defeasible arguments, and $B \nprec_p A$ where $p \in \{\mathtt{Dwl}, \mathtt{Dll}\}$. Then $\exists b \in \mathtt{(L)DR}(B)$, s.t. $\forall a \in \mathtt{(L)DR}(A)$, $b \nless a$.*

Proof. By assumption of $B \not\prec_p A$, and Lemma 2, $(\mathtt{L})\mathtt{DR}(B) \not\triangleleft_{\mathtt{Dem}} (\mathtt{L})\mathtt{DR}(A)$. By Lemma 1, $\neg(\forall b \in (\mathtt{L})\mathtt{DR}(B), \exists a \in (\mathtt{L})\mathtt{DR}(A), b < a)$. That is: $\exists b \in (\mathtt{L})\mathtt{DR}(B)$ s.t. $\forall a \in (\mathtt{L})\mathtt{DR}(A), b \not< a$. □

Lemma 5. *Let A and B be arguments such that* $\mathtt{TopRule}(A) = \mathtt{Conc}(A_1), \ldots \mathtt{Conc}(A_n) \to \psi$, *$A$ is defeasible, and $B \not\prec_p A$. Then for some defeasible A_j, $j = 1 \ldots n$, $B_j \not\prec_p A_j$, where B_j is a strict extension of $\{A_1, \ldots, A_{j-1}, B, A_{j+1}, \ldots, A_n\}$ and $p \in \{\mathtt{Ewl}, \mathtt{Ell}, \mathtt{Dwl}, \mathtt{Dll}\}$.*

Proof. First note that by construction: $(\mathtt{L})\mathtt{DR}(B_j) \cup (\mathtt{L})\mathtt{DR}(A_j) = (\mathtt{L})\mathtt{DR}(B) \cup \bigcup_{i=1}^{n} (\mathtt{L})\mathtt{DR}(A_i)$.

To prove for $p = \mathtt{Ewl}$ ($\mathtt{Ell}$):

Let $\mathcal{E}' = (\mathtt{L})\mathtt{DR}(B), \mathcal{E} = (\mathtt{L})\mathtt{DR}(A)$. $B \not\prec_p A$ and so by Lemma 3:

$$\forall b \in \mathcal{E}', \exists a \in \mathcal{E} \text{s.t.} a \leq b. \tag{1}$$

Suppose some $<$ minimal $a' \in \mathcal{E}$, i.e., $\forall a'' \in \mathcal{E}$, $a'' \not< a'$. Since $\leq$ is a total order: $\forall a'' \in \mathcal{E}$, $a' \leq a''$. Given Eq.1, and transitivity of $\leq$: $\forall b \in \mathcal{E}'$, $a' \leq b$. Hence, $\forall c \in \mathcal{E}' \cup (\mathcal{E} \setminus \{a'\}, c \not< a'$.

Let A_j be the defeasible subargument of A such that $a' \in (\mathtt{L})\mathtt{DR}(A_j)$. Since $(\mathtt{L})\mathtt{DR}(B_j) \subseteq \mathcal{E}' \cup (\mathcal{E} \setminus \{a'\})$, then $\forall c \in (\mathtt{L})\mathtt{DR}(B_j)$, $c \not< a'$. Suppose $(\mathtt{L})\mathtt{DR}(B_j) = \emptyset$. Then by Definition 5 $(\mathtt{L})\mathtt{DR}(B_j) \not\trianglelefteq_{\mathtt{Eli}} (\mathtt{L})\mathtt{DR}(A_j)$, and so $(\mathtt{L})\mathtt{DR}(B_j) \not\triangleleft_{\mathtt{Eli}} (\mathtt{L})\mathtt{DR}(A_j)$. Else if $(\mathtt{L})\mathtt{DR}(B_j) \neq \emptyset$, then by Lemma 1, $(\mathtt{L})\mathtt{DR}(B_j) \not\triangleleft_{\mathtt{Eli}} (\mathtt{L})\mathtt{DR}(A_j)$. By Lemma 2, $B_j \not\prec_p A_j$.

To prove for $p = \mathtt{Dwl}$ ($\mathtt{Dll}$):

– Suppose B is strict, and let $b \in (\mathtt{L})\mathtt{DR}(A)$ be $<$ maximal, i.e., $\forall b' \in (\mathtt{L})\mathtt{DR}(A), b \not< b'$. Let $b \in (\mathtt{L})\mathtt{DR}(A_j)$. If for $i = 1 \ldots n$, $i \neq j$, A_i is strict, then by construction B_j is strict, and so by Proposition 1 (point 2), $B_j \not\prec_p A_j$. Else, let B_j be so constructed so as to include A_j as a sub-argument. Then for any A_i, $i \neq j$, $B_j \not\prec_p A_i$, since $b \in (\mathtt{L})\mathtt{DR}(B_j)$, $\forall b' \in (\mathtt{L})\mathtt{DR}(A_i), b \not< b'$, and so by Lemma 1, $(\mathtt{L})\mathtt{DR}(B_j) \not\triangleleft_{\mathtt{Dem}} (\mathtt{L})\mathtt{DR}(A_i)$, and so be Lemma 2 $B_j \not\prec_p A_i$.

– Suppose B is not strict. By assumption of $B \not\prec_p A$ and Lemma 4, $\exists b \in (\mathtt{L})\mathtt{DR}(B)$ s.t. $\forall a \in (\mathtt{L})\mathtt{DR}(A)$, $b \not< a$. Let A_j be *any* defeasible subargument of A. Since $(\mathtt{L})\mathtt{DR}(B) \subseteq (\mathtt{L})\mathtt{DR}(B_j)$ and $(\mathtt{L})\mathtt{DR}(A_j) \subseteq (\mathtt{L})\mathtt{DR}(A)$, then $\exists b \in (\mathtt{L})\mathtt{DR}(B_j)$ s.t. $\forall a \in (\mathtt{L})\mathtt{DR}(A_j)$, $b \not< a$. By Lemma 1, $(\mathtt{L})\mathtt{DR}(B_j) \not\triangleleft_{\mathtt{Dem}} (\mathtt{L})\mathtt{DR}(A_j)$. By Lemma 2, $B_j \not\prec_p A_j$. □

Lemma 6. *Let A, B and C be defeasible arguments such that $B \not\prec_p A$, $B \prec_p C$. Then $A \prec_p C$ where $p \in \{\mathtt{Ewl}, \mathtt{Ell}, \mathtt{Dwl}, \mathtt{Dll}\}$.*

Proof. To prove for $p = \mathtt{Ewl}$ ($\mathtt{Ell}$):

By assumption of $B \not\prec_p A$ and Lemma 3: **i)** $\forall b \in (\mathtt{L})\mathtt{DR}(B), \exists a \in (\mathtt{L})\mathtt{DR}(A)$ s.t. $a \leq b$.

By assumption of $B \prec_p C$ and Lemma 2, $(\mathtt{L})\mathtt{DR}(B) \triangleleft_{\mathtt{Eli}} (\mathtt{L})\mathtt{DR}(C)$. By Lemma 1: **ii)** $\exists b \in (\mathtt{L})\mathtt{DR}(B), \forall c \in (\mathtt{L})\mathtt{DR}(C), b < c$.

Given **i)**, **ii)** and transitivity of $\leq$, $\exists a \in (\mathtt{L})\mathtt{DR}(A), \forall c \in (\mathtt{L})\mathtt{DR}(C)$, $a < c$. Hence by Lemma 1, $(\mathtt{L})\mathtt{DR}(A) \triangleleft_{\mathtt{Eli}} (\mathtt{L})\mathtt{DR}(C)$, and so by Lemma 2, $A \prec_p C$.

To prove for $p = \mathtt{Dwl}$ ($\mathtt{Dll}$):

By assumption of $B \not\prec_p A$ and Lemma 4, $\exists b \in (\mathtt{L})\mathtt{DR}(B)$ s.t. $\forall a \in (\mathtt{L})\mathtt{DR}(A)$, $b \not< a$. Since $\leq$ is a total order: **i)** $\exists b \in (\mathtt{L})\mathtt{DR}(B), \forall a \in (\mathtt{L})\mathtt{DR}(A), a \leq b$.

By assumption of $B \prec_p C$ and Lemma 2: **ii)** $(\mathtt{L})\mathtt{DR}(B) \triangleleft_{\mathtt{Dem}} (\mathtt{L})\mathtt{DR}(C)$, and so $\forall b \in (\mathtt{L})\mathtt{DR}(B), \exists c \in (\mathtt{L})\mathtt{DR}(C), b < c$.

Given **i)**, **ii)** and transitivity of $\leq$, $\forall a \in (\mathtt{L})\mathtt{DR}(A)$, $\exists c \in (\mathtt{L})\mathtt{DR}(C)$, $a < c$. Hence by Lemma 1, $(\mathtt{L})\mathtt{DR}(A) \lhd_{\mathtt{Dem}} (\mathtt{L})\mathtt{DR}(C)$, and so by Lemma 2, $A \prec_p C$. $\square$

Theorem 4. *Let (Ar, def) be defined by a consistent $(\mathcal{L}, \mathcal{R}, n)$ and $\leq$, and let E be the grounded extension of (Ar, def). Then $\mathtt{Concs}(E)$ is closed under the strict rules, i.e., $\mathtt{Concs}(E) = Cl_S(E)$.*

Proof. It suffices to prove that for an arbitrary $i \geq 1$ it holds that $\mathtt{Concs}(G^i)$ is closed under the set of strict rules. Suppose, towards a contradiction, that this is not the case. That is, there is an $i \geq 1$ such that $\mathtt{Concs}(G^i)$ is not closed under the strict rules. That is, there exists a strict rule $\phi_1, \ldots, \phi_n \rightarrow \psi$ such that $\phi_1, \ldots, \phi_n \in \mathtt{Concs}(G^i)$ but $\psi \notin \mathtt{Concs}(G^i)$. This means that G^i contains arguments $A_1, \ldots, A_n$ with $\mathtt{Conc}(A_1) = \phi_1, \ldots, \mathtt{Conc}(A_n) = \phi_n$ but no argument with conclusion ψ. In particular, G^i does not contain the argument $A_1, \ldots, A_n \rightarrow \psi$ (let's call this argument A).

The fact that argument A is not in G^i implies that it is not strongly defended by G^{i-1}. That is, there exists an argument B that defeats A and that is not strongly defeated by any argument in G^{i-1}. The only way B can defeat A is by rebutting A's top-conclusion, because otherwise B would also defeat an argument in $\{A_1, \ldots, A_n\}$, which would imply that these are not strongly defended by G^{i-1} (and therefore could not be in G^i). Since $(\mathcal{L}, \mathcal{R}, n)$ is consistent, it cannot be that both A and B are strict. Hence, if A is strict, then B must be defeasible, but then by Proposition 1 (point 1), $B \prec_p A$, contradicting B defeats A. Hence it must be that A is defeasible. By Lemma 5, and the fact that the strict rules are closed under transposition, one can construct an argument $B_j = A_1, \ldots, A_{j-1}, B, A_{j+1}, \ldots, A_n \rightarrow \neg\phi_j$ which rebuts A_j, such that $B_j \not\prec_p A_j$. Hence B_j defeats A_j.

Since $A_j \in G^i$, G^{i-1} strongly defends A_j. This means that G^{i-1} contains an argument (say, A'_j) that strongly defeats B_j. A'_j cannot strongly defeat B (recall that B was assumed to have no strong defeaters in G^{i-1}) and cannot defeat any argument in $\{A_1, \ldots, A_{j-1}, A_{j+1}, \ldots, A_n\}$ (otherwise, the fact that $G^{i-1} \subseteq G^i$ would imply that $A'_j \in G^i$, so G^i is not conflict-free, so $E = \cup_{i=0}^{\infty} G^i$ is not conflict-free, which contradicts the fact that E is the grounded extension). Therefore, A'_j must strongly defeat B_j on its top-conclusion. That is, $\mathtt{Conc}(A'_j) = \phi_j$. The fact that A'_j strongly defeats B_j, implies that $B_j \prec_p A'_j$. From this and the earlier obtained fact that $B_j \not\prec_p A_j$ it follows (Lemma 6) that $A_j \prec_p A'_j$.

Consider the case where we take argument A and replace (substitute) its subargument A_j by the stronger A'_j (this is possible because both A_j and A'_j have the same conclusion ϕ_j). Call the resulting argument A'. Now, if A' in turn has a defeater (say B') that is not strongly defeated by G^{i-1}, then we can use the same line of reasoning as above to identify subargument of A' that can be replaced by a stronger subargument with the same conclusion. This would lead to a sequence of arguments $A', A'', A''', \ldots$ in which a sequence of subarguments is replaced by stronger subarguments with the same conclusion.

Does such a sequence ever terminate, or can it go on infinity? Suppose, towards a contradiction, that there exists an infinite sequence $A', A'', A''', \ldots$. Since the number of arguments in the argumentation framework is finite (more precisely: the number of arguments with a particular conclusion is finite) it holds that sooner or later, one has to run out of of "fresh" (unused) subarguments and substitute a subargument in that was previously substituted out. Let us call this argument A_{k_1}, with $\mathtt{Conc}(A_{k_1}) = \phi_k$. Now, in the sequence of substituted subarguments, let us specifically look for those subarguments with conclusion ϕ_k. So, at some moment, A_{k_1} was substituted by some A_{k_2}, after which possibly some other subarguments were substituted, after which A_{k_2} was substituted by A_{k_3}, after which possibly some other subarguments were substituted, etc, until at some moment A_{k_m} was substituted by A_{k_1}. Now, the fact that A_{k_1} was substituted by A_{k_2} means that (as we have seen before) $A_{k_1} \prec_p A_{k_2}$. The fact that A_{k_2} was substituted by A_{k_3} means that $A_{k_2} \prec_p A_{k_3}$, etc. So from transitivity, we obtain that $A_{k_1} \prec_p A_{k_m}$. But the fact that A_{k_m} was substituted by A_{k_1} means that $A_{k_m} \prec_p A_{k_1}$. Contradiction.

The fact that each sequence of substitutions has to be finite means that the sequence $A, A', A'', A''', \ldots$ has to terminate at some point with an argument (say, A^*) that is not eligible for substitution anymore. This means that A^* cannot have a defeater (say B^*) that isn't strongly defeated by G^{i-1} (because if it did have such a defeater then, using the reasoning earlier in this proof, we could substitute one of its subarguments). This means that A^* is strongly defended by G^{i-1}. Therefore, $A^* \in G^i$. But that would mean that $\psi \in \texttt{Concs}(G^i)$ (since $\psi = \texttt{Conc}(A^*)$), which contradicts our initial assumption that $\psi \notin \texttt{Concs}(G^i)$. □

Finally, we show that *indirect consistency* holds; that is, the closure under strict rules of the grounded extension is consistent.

Theorem 5. *Let* (Ar, def) *be defined by a consistent* $(\mathcal{L}, \mathcal{R}, \texttt{n})$ *and* $\leq$*, and let* E *be the grounded extension of* (Ar, def). $\texttt{Concs}(E)$ *satisfies* indirect consistency*:* $\neg\exists\alpha$ *such that* $\alpha, \neg\alpha \in Cl_S(E)$.

Proof. Follows immediately from Theorem 1 and Theorem 4. □

5. Discussion and Conclusions

This paper has presented a modified version of the ASPIC+ framework that applies unrestricted rebut in combination with various widely used preference principles. We showed that the resulting ASPIC- formalism satisfies [2]'s rationality postulates when applying grounded semantics and under the assumption that the preferences are based on a total ordering over the defeasible rules. Of course, one limitation is that unlike [2,12], the rationality postulates are not shown to hold for all complete-based semantics (that is, a semantics whose extensions are complete). Broadening the unrestricted rebut approach to semantics other than grounded is far from trivial (a counter example against consistency under preferred semantics is given in [7]). Also, note that problems with applying unrestricted rebut with anything other than the grounded semantics are not restricted to ASPIC, or even to rule-based argumentation in general, as similar difficulties have been observed in logic-based argumentation [10]. Another restriction of our approach is the assumption of a total order on defeasible rules. However, our preliminary investigations show that if, given a partial ordering, preferences are defined only if they are sanctioned by all total orderings that preserve the given partial ordering, then the rationality postulates can still be shown to hold. We will pursue this line of research in future work.

Given our results and [2], we can conclude that if one wants to have flexibility on which semantics to use, one has to apply restricted rebut, with its possible counter-intuitive behaviour in dialectical settings. Switching to unrestricted rebut, on the other hand, does allow for more natural dialogues,[6] but (unless the results of the current paper can somehow be broadened) also restricts one to use grounded semantics. Thus, the choice of whether to apply restricted or unrestricted rebut depends on the intended application. Apart from describing how unrestricted rebut can be used, the current paper also helps to highlight what the advantages and disadvantages of each approach are, so that those interested in applying argumentation formalisms can make an informed choice.

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[6] Which is a key objective for realising the long-term research goal of adding dialectical aspects to Dung-style instantiated argumentation, in a manner that provides for "mixed initiative" argumentation in which human users can participate and contribute to computational argumentation-based reasoning.